Stochastic Approach to Inspection Evaluation: Methodology and Validation

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Abstract

The paper discusses the methodology for performing International Atomic Energy Agency's (IAEA's) post-inspection analysis to assess the effectiveness of verification inspection plans using a stochastic method. Conventionally, well-established statistical distributions are employed to calculate Detection Probability (DP) which is the effectiveness metric for both planning and evaluation purposes. The detection probability here is the probability of detecting at least one "defective" item (an item from which material has been removed) from the multi-defect sample space of items. The DP, in turn depends on the probability that a defected item is randomly selected for measurement (the "selection probability") and the probability that the applied measurement identifies the defect (the "identification probability"). The stochastic method described here involves simulating the inspection process by randomly choosing a fixed number of items from a population of items and performing measurements for these samples. A detection probability value is calculated at the end of a simulation depending on the random outcome. Multiple such simulations/trials are performed on the same sample space to get multiple detection probabilities. The Final Detection Probability and its uncertainty are estimated by computing the average and standard error of all the DP values from all simulations. The stochastic model development, its verification, and benchmarking are discussed in detail.

Keywords: Stochastic Approach; Detection Probability; Selection Process

1. Introduction

The increase in computational power in modern computers and the development of pseudorandom generators have resulted in the prevalent use of state-of-the-art Monte-Carlo/Stochastic methods [1] for many applications. These methods allow us to harness the computational power to simulate real-world experiments involving probabilities and random processes. With known outcomes and outcome probability distribution function (pdf) [2], any random process can be simulated by invoking a pseudorandom generator satisfying the required pdf function. The outcomes simulated by the random generator can contain related and unrelated events to our quantity of interest. For example, in the coin-toss experiment, if the probability of getting Heads P(Heads) is our quantity of interest, then Head events are related events, and Tail events are unrelated to our quantity. Such quantities of interest that are involved in the process can be derived/computed based on the relative frequency with which the random generator simulates the quantity-related events. The true power of stochastic methods becomes apparent when dealing with complex random processes which contain multiple simple random processes embedded within the complex process. Such complex processes can be simulated by invoking multiple pseudorandom number generators, with each generator simulating one of the embedded simpler random processes. The entire complex process can be simulated by concatenating the outputs of one simple process with the input of another simple process. The real-world inspection problem is an example of such a complex random process. It has a random selection process followed by an instrumental measurement process embedded sequentially. The detection probability DP is the primary quantity of interest. In further sections, we shall describe the conventional way of deterministically evaluating DP using distributions and evaluating DP from stochastic simulations.

A probabilistic model of the IAEA's inspection problem [3] is that of random selection from a set of identical items, from some of which a proliferator has removed some amount of material. Items from which material has been removed are referred to as defects or defective items. The original set of items following proliferation, in general, contains both defects and non-defects. Depending on the proliferator's diversion strategy, multiple types of defects (each type of defect is resulted from removing different amounts of material from the original item) can be induced in the sample space. For example, consider a sample space or a stratum containing ten items; following diversion, two of the items are transformed into Defects, and the rest remain unchanged (ND). Among the two defects, assume both are different types of Defects (D_1, D_2).

Sample space: {D_1, D_2, ND} = [1, 1, 8]; Total = 10 items.

In the remainder of this section, we illustrate a deterministic approach to calculating the probability of detecting diversion using the above example. In sections 2 & 3, we describe the stochastic approach and demonstrate its application to examples, including validation of the approach against a previously published deterministic solution.

1.1 Illustration of a Deterministic Approach

The inspection process involves randomly selecting a few items and performing measurements on the selected items using an instrument (method) from a range of choices, each with a unique measurement fidelity and uncertainty. The instrument or method's ability to detect a specific item in the sample space as a defect varies with the type of item being measured, characterized as a probability that the measurement method identifies a defective item. This probability is termed Identification probability (IP). Assume that the instrument identifies D_1 items 100% of the time as defected, D_2 items 50% of the time, and the measurement never identifies non-defects as defected, i.e., IP = 0%.

[IP_{D1}, $IP_{D2,}$ IP_{ND}] = [1, 0.5, 0]

Analytically, the overall DP is computed by summing up individual DP components corresponding to all possible outcomes of the random selection of the set of items in the sample space. For each outcome, its DP value is given by the product of the outcome's selection probability (SP) and identification probability (IP). The conditional tree diagram in Figure 1 exemplifies the identification of all possible selection outcomes and the determination of each outcome's selection probability.

For Single Measurement Inspection, a single item is randomly sampled for measurement. The left conditional tree diagram in Figure 1 shows three possible outcomes of single measurement sampling where one of the three item types will be selected. Therefore, for a single measurement inspection, the Detection Probability DP is given by the sum of component DPs of all outcomes.

$DP = SP_{D1}*IP_{D1} + SP_{D2}*IP_{D2} + SP_{ND}*IP_{ND} = (1/10)*1 + (1/10)*0.5 + (8/10)*0 = 0.15 = 15\%$.

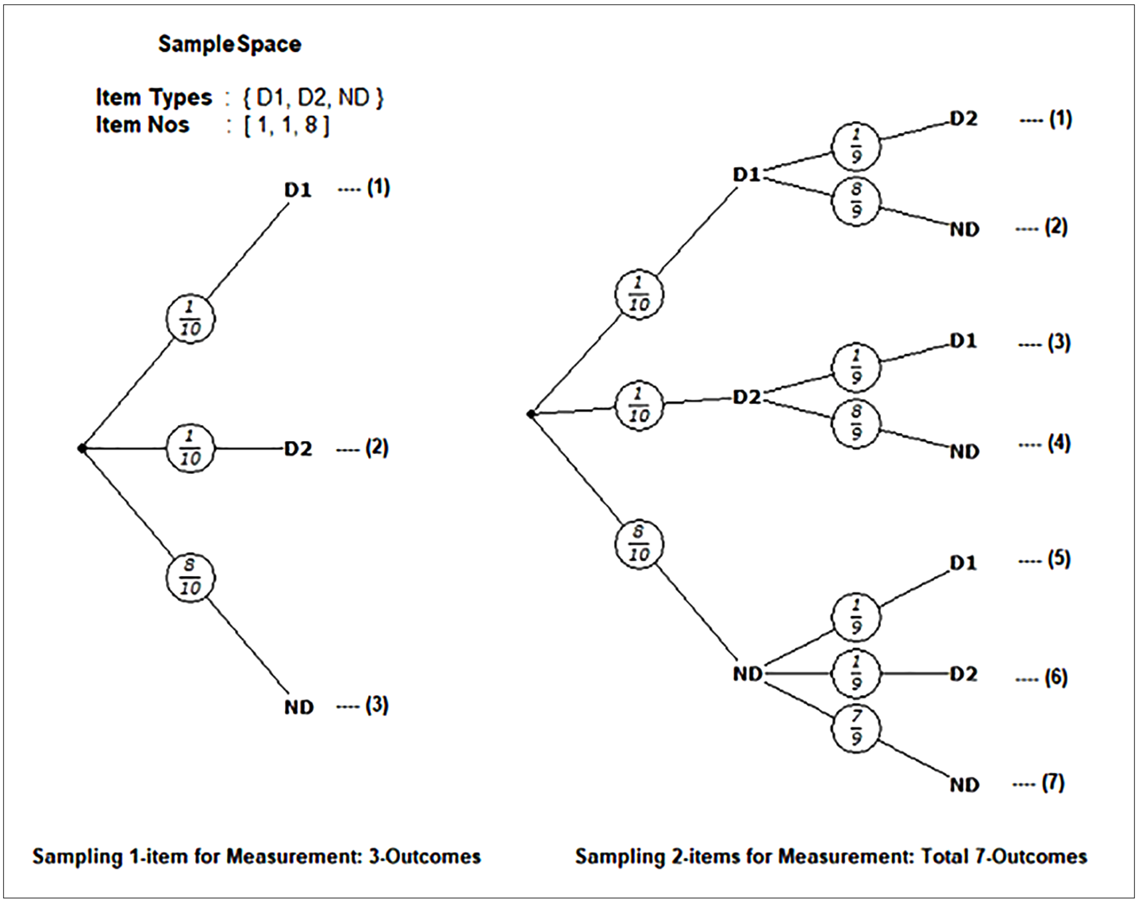

Figure 1: Conditional Tree Diagrams Depicting Various Outcomes and their Selection Probabilities.

For Double Measurement inspection, two items are randomly sampled for measurement. There are seven possible combinations described in the right conditional tree diagram in Figure 1. As the number of item types and measurements increases, the total combinations of selection outcomes required to evaluate DP quickly gets large. For a generic data set of items N and number of measurements n the multivariate hypergeometric PDF (denoted *'MVHG_PDF'* below) is used to compute selection probabilities of different combinations of outcomes as shown below. The MVHG_PDF gives the conditional probability of n draws, without replacement, from a finite population of size N that contains i types of items with I_i numbers in the population leading to the selection of x_i numbers of respective item types in the outcome upon sampling.

$$SP(x_1, x_2, x_3 \dots, x_i, I_1, I_2, I_3 \dots, I_i, N, n) = MVHG\_PDF$$

$$= \frac{\binom{I_1}{x_1}\binom{I_2}{x_2}\binom{I_3}{x_3}\dots\binom{I_i}{x_i}}{\binom{N}{n}}$$

$$DP(x_1, x_2, x_3 \dots) = SP(x_1, x_2, x_3, I_1, I_2, I_3 \dots, N, n) * IP$$

$$= \frac{\binom{I_1}{x_1}\binom{I_2}{x_2}\binom{I_3}{x_3}\dots\binom{I_i}{x_i}}{\binom{N}{n}} * \left[1 - \prod NIP_i^{x_i}\right]$$

$$TDP = \sum_{All\ combinations} DP(x_1, x_2 \dots, x_i)$$

$$= \sum_{All\ combinations} \frac{\binom{I_1}{x_1}\binom{I_2}{x_2}\binom{I_3}{x_3}\dots\binom{I_i}{x_i}}{\binom{N}{n}} * \left[1 - \prod NIP_i^{x_i}\right]$$

The variables and constants used in the equations above are as follows:

i = 3 (Three item types in stratum {D_1, D_2, ND})

I_1 = Total number of D1 items = 1

I_2 = Total number of D2 items = 1

I_3 = Total number of ND non-defect items = 8

N = Total number of items in stratum = $I_1 + I_2 + I_3$ = 10

n = Number of items randomly sampled from total items for inspection = 2

x_1 = Number of D_1 items in sample

x_2 = Number of D_2 items in sample

x_3 = Number of ND items in sample

NIP_i = non-identification probability of i^{th} item type = 1 - IP_i

TDP = Total detection probability

The terms $\binom{a}{b}$ represent typical combination operation $C(a,b) = \frac{a!}{(a-b)!\ b!}$

Note that x_1, x_2, and x_3 values vary for different outcomes or combinations.

Calculation of Selection Probabilities:

- [D_1 D_2] and [D_2 D_1] combination: $SP = \frac{\binom{I_1}{x_1}\binom{I_2}{x_2}\binom{I_3}{x_3}}{\binom{N}{n}} = \frac{\binom{1}{1}\binom{1}{1}\binom{8}{0}}{\binom{10}{2}} = \frac{1}{45}$
- [D_1 ND] and [ND D_1] combination: $SP = \frac{\binom{I_1}{x_1}\binom{I_2}{x_2}\binom{I_3}{x_3}}{\binom{N}{n}} = \frac{\binom{1}{1}\binom{1}{0}\binom{8}{1}}{\binom{10}{2}} = \frac{8}{45}$
- [D_2 ND] and [ND D_2] combination: $SP = \frac{\binom{I_1}{x_1}\binom{I_2}{x_2}\binom{I_3}{x_3}}{\binom{N}{n}} = \frac{\binom{1}{0}\binom{1}{1}\binom{8}{1}}{\binom{10}{2}} = \frac{8}{45}$

[ND ND] combination: $SP = \frac{\binom{I_1}{x_1}\binom{I_2}{x_2}\binom{I_3}{x_3}}{\binom{N}{n}} = \frac{\binom{1}{0}\binom{1}{0}\binom{8}{2}}{\binom{10}{2}} = \frac{28}{45}$

The usage of the multivariate hypergeometric distribution multiple times is necessary to account for selection probabilities for various possible outcomes. With the increase in the number of measurements and item types in the sample space, the inspection outcomes increase exponentially. This exponential increase in inspection outcomes quickly limits the model's performance in terms of computational resources (CPU and Memory). The development, performance and limitations of conditional tree-based deterministic models will be discussed extensively in a forthcoming paper [9]. The illustration of DP calculations based on a deterministic approach shows how the calculation can quickly become rather complicated (in terms of identifying outcomes) even for a single stratum of material, let alone multiple strata within a facility and ultimately multiple facilities within a state. So far, the examples depicted in Figure (1) use a single instrument or measurement method, and it must be noted that the deterministic models get even more complicated in multi-instrument inspections. This is why

Combination Type	Selection Probability	Non-Detection Probability
[D_1 D_2] and [D_2 D_1]	$\frac{1}{45}$	$\frac{1}{45} * (1-1) * (1-0.5) = 0$
[D_1 ND] and [ND D_1]	$\frac{8}{45}$	$\frac{8}{45} * (1-1) * (1-0) = 0$
[D_2 ND] and [ND D_2]	$\frac{8}{45}$	$\frac{8}{45} * (1-0.5) * (1-0) = \frac{4}{45}$
[ND ND]	$\frac{28}{45}$	$\frac{28}{45} * (1-0) * (1-0) = \frac{28}{45}$
Total Non-Detection Probability		$0 + 0 + \frac{4}{45} + \frac{28}{45} = 0.711112$
Total Detection Probability		$1 - (0.71112) = 0.28889 = \mathbf{28.89\%}$

Table 1: Calculation of Total Detection Probability for Double-item Measurement Inspection

the deterministic models developed in literature are case-specific and lack universal applicability.

A stochastic approach provides an intuitive and flexible alternative. It involves simulating the inspection process, randomly selecting items from the stratum followed by instrumental measurements on each selected item, and repeating this simulation multiple times to acquire a distribution of DP values. The mean of this distribution of simulated DP values and its standard error provide estimates of the total DP and its uncertainty, respectively. The accuracy of the result increases with the increase in the number of simulated inspections. Individual inspection simulations require low computer memory requirements relative to the deterministic approach. Individual simulations are independent of each other, so the increase in computational cost is primarily in terms of CPU, which is easily manageable on a generic multi-threading and multi-core computer. We discuss the stochastic approach in detail in the next section.

2. Stochastic Approach

The stochastic approach uses a set of random simulations or trials, called an ensemble, to generate a distribution of outcomes from which the best estimate of the desired quantity is computed. The stochastic nomenclature used in this paper is summarised below:

- *Stochastic Simulation/Trial:* A single (pseudo-) random sample of a random variable or process.
- *Outcome:* A possible result of a simulation or trial.
- *Ensemble:* A set of outcomes acquired from multiple simulations or trials.
- *Ensemble Mean:* The mean of an ensemble (when outcomes are numerical values).
- *Stochastic Solution:* An estimate of the desired quantity acquired from ensemble means (this may involve multiple ensembles and is computed from the average of all ensemble means).
- *Stochastic Standard Error:* Standard error in the estimated stochastic solution computed using the ensemble means.

A simple example problem illustrates the application of the stochastic approach. Suppose we simulate an unbiased coin-toss experiment where the outcomes are Heads or Tails. To estimate the probability of getting "Heads" on a single toss, the act of tossing is simulated many times using a uniform pseudorandom number generator which yields a value of 0 or 1. In each simulation or trial, a number is sampled from the pseudorandom generator; getting 1 is equivalent to getting Heads, and 0 means Tails, respectively. Since we are looking for Head events, we assign the value 1 to the stochastic outcome when a Head turns up. If a Tail turns up, we assign 0 to the stochastic outcome. We simulate the experiment 100 times (collectively called an ensemble) and store the outcomes as 0s and 1s. The ensemble mean or the mean of the outcomes of 100 trials will yield a value close to 0.5 with an estimate of error associated with the result. By increasing the number of simulations/trials, the ensemble mean will get closer to 0.5. Theoretically, the value will converge to 0.5 with zero error with an infinite number of trials. Thus, using the stochastic approach, the probability of the outcome "Heads" is estimated by repeatedly simulating a coin toss using a pseudorandom number generator multiple times, counting the occurrence of "Heads" and dividing by the total number of simulations (or, equivalently, averaging the numerical values assigned to Heads and Tails) represents the stochastic solution. In the following sub-section, we describe the application of the stochastic process to evaluate the effectiveness of the IAEA inspection using pseudorandom generators.

2.1 Methodology for Inspection Problem

Applying the stochastic approach to inspection involves simulation of the random selection of a specified number of items from the set of all possible items, followed by measurements on selected items. For each simulation, a DP value (the outcome) is calculated. The simulation is repeated multiple times to acquire a sufficient distribution of DP values (the ensemble). The mean of this distribution is the desired approximation to DP (the stochastic solution) for the specified inspection campaign data.

2.1.1 Selection Process

Consider the single-item and double-item inspection examples discussed in Section 1, where items are randomly selected from the set of one D1, one D2, and eight ND items. Consider $M \times N$ inspection simulations representing M independent ensembles of N trials each. For practical reasons, it is convenient to split the total number of trials into multiple ensembles. Figure 2 describes the outcomes of $M \times N$ simulations for single- and double-item inspection examples.

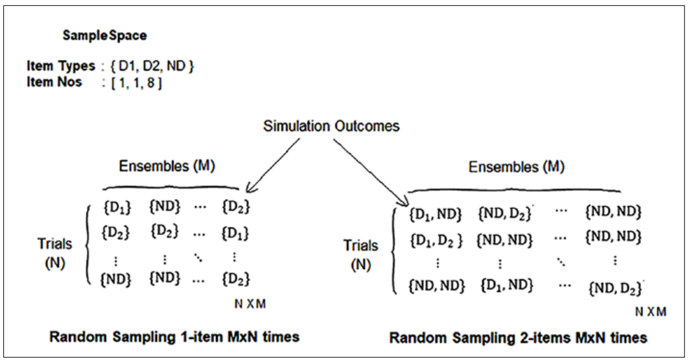

Figure 2: Random Selection Matrices Depicting Various Outcomes and their Event spaces.

2.1.2 Identification Process

The second step of the inspection process involves measuring the selected items. Measuring selected items will allow the inspector to identify defects. Using the same identification probabilities as in the deterministic treatment, [IP_{D1}, IP_{D2}, IP_{ND}] = [1, 0.5, 0] gives identification probabilities and NIP = 1-IP gives the non-identification probabilities. The identification step involves replacing the item types within the simulation matrices with their overall non-identification probabilities. The items in Figure 2 are replaced by their non-identification probabilities to get Figure 3. Then the overall outcome identification probabilities are computed in Figure 4 by multiplying item NIPs within all brackets present in Figure 3.

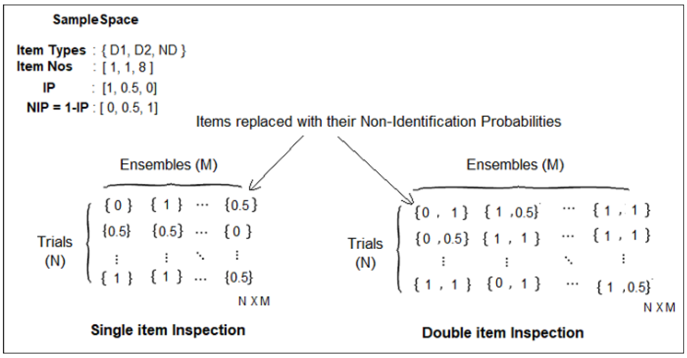

Figure 3: Non-Identification Probability Matrices

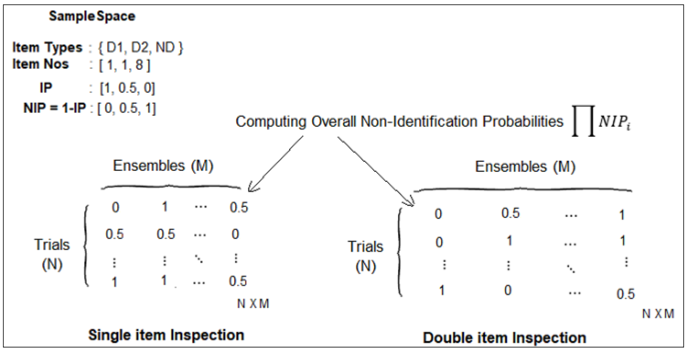

Figure 4: The Overall Outcome Non-Identification Probability Matrices

2.1.3 Computing Detection Probability DP

To compute the overall DP for a specific inspection campaign, we must first calculate the non-detection probability (NDP) corresponding to each simulated inspection. The non-identification probability (NIP) value for each outcome shown in Figure 4 is, in fact, the NDP value for the respective outcome. The detection probability is 1 - NDP, as shown in Figure 5. For each ensemble of N trials, our implementation of the stochastic approach computes an ensemble mean DP and standard error. The approach then computes the aggregate mean and standard error over the M ensembles. We shall discuss all the necessary derivation steps to get to the final DP estimate and its standard error in detail in section 2.2.

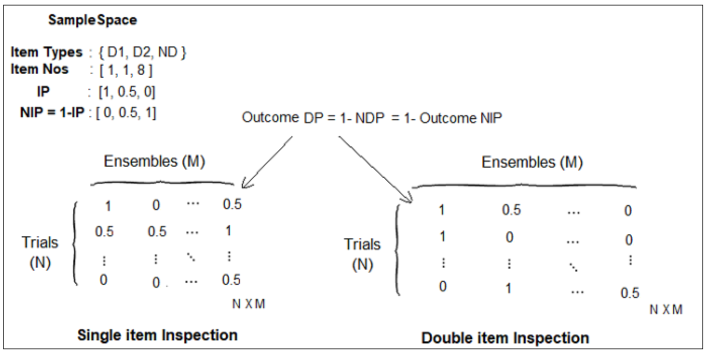

Figure 5: The Outcome Detection Probability Matrices

2.2 Stochastic Standard Error

Let X_{ij} represent DP for the ith simulation and j[th] ensemble. The values for 'i' range from 1 to N trials, and 'j' takes values from 1 to M ensembles. Assume that all DP values are independent and identically distributed random variables with mean μ and variance σ^2. The complete set of DP values from Figure 5 can be represented in terms of the following matrix:

$$\begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1M} \\ X_{21} & X_{22} & \cdots & X_{2M} \\ \vdots & \vdots & X_{ij} & \vdots \\ X_{N1} & X_{N2} & \cdots & X_{NM} \end{pmatrix}$$

2.2.1 Overall Statistics

The overall estimate of DP and uncertainty can, in principle, be calculated by the simple statistical formulas below:

$$\hat{\mu} = \bar{X}_{..} = \frac{1}{MN}\sum_{ij} X_{ij}$$

$$s^2 = \frac{1}{MN-1}\sum_{ij}\left(X_{ij} - \bar{X}_{..}\right)^2 \qquad (1)$$

$$se(\hat{\mu}) = \frac{s}{\sqrt{MN}}$$

Here we make use of the typical statistical notation: $\hat{\mu}$ denotes the estimated mean, which is the average of all DP values, $\bar{X}_{..}$; s denotes sample standard deviation, and se denotes standard error in the estimated parameter. However, we first calculate these statistics for individual ensembles of N DP values and then aggregate them across ensembles to estimate a final DP value and uncertainty. The reason for breaking the entire collection of outcomes into separate ensembles has to do with our lack of knowledge of the number of simulations/trials needed to achieve a target convergence criterion prior to simulations. It is computationally convenient to run one ensemble of N simulations/trials at a time, estimate running standard error, and decide based on the acquired error whether to run further ensembles or not. The following sub-sections consider the statistics for individual ensembles and across the ensembles of simulation averages.

2.2.2 Ensemble Statistics

Each ensemble consists of N trials that yield N DP values. For the j^{th} ensemble, the mean, standard deviation, and standard error in the mean are as follows:

$$\hat{\mu}_j = \bar{X}_{.j} = \frac{1}{N}\sum_i X_{ij}$$
$$s_j^2 = \frac{1}{N-1}\sum_i \left(X_{ij} - \bar{X}_{.j}\right)^2 \quad (2)$$
$$se\left(\hat{\mu}_j\right) = \frac{s_j}{\sqrt{N}}$$

2.2.3 Statistics Across Ensembles

Aggregate mean, standard deviation, and standard error across the ensembles may be computed as follows:

$$\hat{\mu}_{avg} = \bar{X}_{..} = \frac{1}{M}\sum_j \bar{X}_{.j} = \hat{\mu}$$
$$s_{avg}^2 = \frac{1}{M-1}\sum_j \left(X_{.j} - \bar{X}_{..}\right)^2 \quad (3)$$
$$se\left(\hat{\mu}_{avg}\right) = \frac{s_{avg}}{\sqrt{M}}$$

2.2.4 Overall Statistics: Combining Ensemble & Across Ensemble Statistics

The breakdown of the complete set of simulations into multiple subsets of ensembles is similar to the "within-group" and "across-group" calculations used in the analysis of variance (ANOVA) [5]. We apply the same calculations used in ANOVA to compute overall statistics. First, the overall mean is simply the average of ensemble means, as noted in Equation (3). The standard error is computed as follows [6]:

$$s^2 = \frac{1}{MN-1}\sum_{ij}\left(X_{ij} - \bar{X}_{..}\right)^2 \rightarrow from\ (1)$$

$$s^2 = \frac{1}{MN-1}\sum_{ij}\left(X_{ij} - X_{.j} + X_{.j} - \bar{X}_{..}\right)^2$$

$$s^2 = \frac{1}{MN-1}\sum_{ij}\left[\left(X_{ij} - X_{.j}\right)^2 + 2\left(X_{ij} - X_{.j}\right)\left(X_{ij} - X_{.j}\right) + \left(X_{.j} - \bar{X}_{..}\right)^2\right]$$

$$s^2 = \frac{1}{MN-1}\left[\sum_{ij}\left(X_{ij} - X_{.j}\right)^2 + 0 + N\sum_j\left(X_{.j} - \bar{X}_{..}\right)^2\right]$$

$$s^2 = \frac{1}{MN-1}\left[\sum_j (N-1)s_j^2 + 0 + N(M-1)s_{avg}^2\right] \rightarrow from\ (2)\ \&\ (3)$$

$$s^2 = \frac{M(N-1)\overline{s_{.}^2} + N(M-1)s_{avg}^2}{MN-1}$$

$$s^2 = \frac{M(N-1)\overline{s_{.}^2}}{MN-1} + \frac{N(M-1)s_{avg}^2}{MN-1}$$

Converting Variances into Standard errors,

$$MNse(\hat{\mu})^2 = \frac{M(N-1)N\,\overline{se\left(\hat{\mu}_j\right)^2}}{MN-1} + \frac{N(M-1)M\,se\left(\hat{\mu}_{avg}\right)^2}{MN-1}$$

$$se(\hat{\mu})^2 = \frac{(N-1)\,\overline{se\left(\hat{\mu}_j\right)^2}}{MN-1} + \frac{(M-1)\,se\left(\hat{\mu}_{avg}\right)^2}{MN-1}$$

$$se(\hat{\mu}) = \sqrt{\frac{(N-1)\,\overline{se\left(\hat{\mu}_j\right)^2}}{MN-1} + \frac{(M-1)\,se\left(\hat{\mu}_{avg}\right)^2}{MN-1}} \quad (4)$$

$$where\ \overline{se\left(\hat{\mu}_j\right)^2} = \frac{1}{M}\sum_j se\left(\hat{\mu}_j\right)^2$$

$$\hat{\mu} = \frac{1}{M}\sum_j \bar{X}_{.j} = \frac{1}{MN}\sum_{ij} X_{ij} \quad (5)$$

The Overall Average $\hat{\mu}$ from Equation (5) gives the best estimate of Detection Probability, with Equation (4) as the best estimate of its standard error $se(\hat{\mu})$. Based on the described stochastic approach, a python model has been developed. The model allows users to input the required standard error in DP estimate, trials per ensemble, and case data. The model starts with a single ensemble of stochastic simulations and computes DP & standard error using equations (4) & (5). It initiates a new ensemble of stochastic simulations, recomputes running error, and repeats the process until the running standard error converges to a user-set value. The single-item & double-item inspection examples are simulated using the stochastic model with the required error set to 0.002, and the number of trials per ensemble *N* is set to 2000. For the single-item inspection example, the code ran 13 ensembles, and the final DP value is 0.148 with 0.002 as the standard error in the estimate. For the double-item inspection case, the code ran 21 ensembles, and the final DP value is 0.287 with 0.002 as the standard error in the estimate. By comparison, the stochastic results agree with the deterministic results in section 1.1, i.e., DP is 0.15 for the single-item inspection, and DP is 0.289 for the double-item inspection.

3. Validation of the Stochastic Approach

In the publication Krieger et al. [7], the authors investigate scenarios to develop inspection sampling plans for inventory verification of spent fuel ponds. The paper discusses probable diversion scenarios from the spent fuel storage ponds and calculates the achieved DP for the specified sampling plans. We choose this paper primarily as it defines various inspection scenarios, treats them deterministically, and computes DP, all in one place, sufficient for our benchmarking purposes. We calculate the DP for two cases mentioned in the paper [7] using our stochastic approach [4] and compare our results to the published results [7, 8].

3.1 Example: Varying Falsified Pins

In this example, the spent fuel pond contains 2500 (*N*) spent fuel assemblies (SFAs), with each assembly containing 96 (*L*) fuel pins. In terms of material, each assembly contains 2 kg or 0.25 SQ ($\bar{x}$) of Pu. A total goal amount (G) of 1 SQ or 8 Kg of Pu is chosen to be diverted by removing r_{pins} from each assembly. To acquire 1 SQ would require r_{SFA} assemblies from which r_{pins} pins are removed while the remaining N-r_{SFA} assemblies remain untouched. The r_{pins} falsified pins per assembly are varied from 1 to 96 in steps of 1. The total number of assemblies r_{SFA} required to divert 1 SQ is given by equation (6).

$$\boldsymbol{r_{pins}} = [\mathbf{1, 2, 3, \ldots, 95, 96}]$$

$$\boldsymbol{r_{SFA}(r_{pins})} = \boldsymbol{ceil}\left(\frac{\boldsymbol{G * L}}{\boldsymbol{\bar{x} * r_{pins}}}\right) \quad (6)$$

Out of 2500 SFAs, the inspector verifies n_1 SFAs with the ICVD, n_2 SFAs with the DCVD, and n_3 SFAs with the PGET, where per verified SFA only one measurement instrument is applied. For the given example, the values of n_1, n_2, and n_3 are taken to be 10, 65 & 25 measurements, respectively.

Each instrument's identification probability function is modeled as a step function; i.e., the identification probability is 0 or 1 when the number of pins diverted in a measured assembly is less than or greater than a certain % of total pins, respectively, as shown in Equation (7). The ICVD detects diversion only when 100% of pins are absent from the measured SFA. DCVD detects diversion when 30% of total pins are absent. PGET detects diversion when 0.38% of total pins are missing from the measured SFA. Therefore, the piece-wise function in Equation (7) gives the instrument identification probability.

$$IP(r_{pins}, L, \%Limit) = \begin{matrix} 1 \; if \; \left(\frac{r_{pins}}{L} * 100\right) >= \%Limit \\ 0 \; if \; \left(\frac{r_{pins}}{L} * 100\right) < \%Limit \end{matrix} \quad (7)$$

The following summarizes Equation (7) and the number of measurements for each instrument type:

$$Using\ ICVD, \%Limit = 100;\ IP_{ICVD} = IP(r_{pins}, L, 100);\ n_1 = 10$$
$$Using\ DCVD, \%Limit = 30;\ IP_{DCVD} = IP(r_{pins}, L, 30);\ n_2 = 65$$
$$Using\ PGET, \%Limit = 0.38;\ IP_{PGET} = IP(r_{pins}, L, 0.38);\ n_3 = 25$$

The overall DP for this example has been discussed in Krieger et al. [7]. The piece-wise DP equation from [7] is repeated below.

$$\mathrm{DP}(\mathrm{N}, \mathrm{n_1}, \mathrm{n_2}, \mathrm{n_3}, \mathrm{r_{pin}}) = \begin{cases} 1 - \dfrac{\binom{\mathrm{N} - \mathrm{r_{SFA}}(\mathrm{r_{pins}})}{\mathrm{n_3}}}{\binom{\mathrm{N}}{\mathrm{n_3}}} & \text{for } \mathrm{r_{pin}} < [0.3\mathrm{L}] - 1 \\ 1 - \dfrac{\binom{\mathrm{N} - \mathrm{r_{SFA}}(\mathrm{r_{pins}})}{\mathrm{n_2} + \mathrm{n_3}}}{\binom{\mathrm{N}}{\mathrm{n_2} + \mathrm{n_3}}} & \text{for } 0.3\mathrm{L} < \mathrm{r_{pin}} < \mathrm{L} - 1 \\ 1 - \dfrac{\binom{\mathrm{N} - \mathrm{r_{SFA}}(\mathrm{r_{pins}})}{\mathrm{n_1} + \mathrm{n_2} + \mathrm{n_3}}}{\binom{\mathrm{N}}{\mathrm{n_1} + \mathrm{n_2} + \mathrm{n_3}}} & \text{for } \mathrm{r_{pin}} = \mathrm{L} \end{cases} \quad (8)$$

We applied the stochastic approach to compute the overall detection probability and uncertainty with Equations (4) and (5). We used 2000 trials per ensemble with a target standard error set to 0.002. The code automatically generates ensembles until the running standard error is less than or equal to the set value. Convergence in standard error with number of ensembles to the set value is illustrated in Figure (6). The plot demonstrates how different falsified pin examples converge at a different rate to the set error and also shows the practicality of estimating running standard error that allowed the code to stop initiating additional ensembles when the error reaches the set value. The deterministic and stochastic results are co-plotted in Figure (7), showing the agreement between both. The same agreement is further depicted in the residual plot of Figure (8), where the difference between deterministic and stochastic DPs are computed and plotted along with set standard error limits. All the residual values plotted in Figure (8) lie within the limits of three times set standard error, indicating the agreement of the stochastic results with that of deterministic results.

The lowest detection probability occurs at r_{pins} = 28. The deterministic estimate of DP is 0.1315, while the estimate using the stochastic approach is 0.1309 with 0.0019 as its standard error. The residual between deterministic and stochastic DP estimates is 0.0006, which lies within the limits of twice the stochastic standard error (± 2*SE), i.e., ± 0.0038. Therefore, the stochastic results agree with deterministic results.

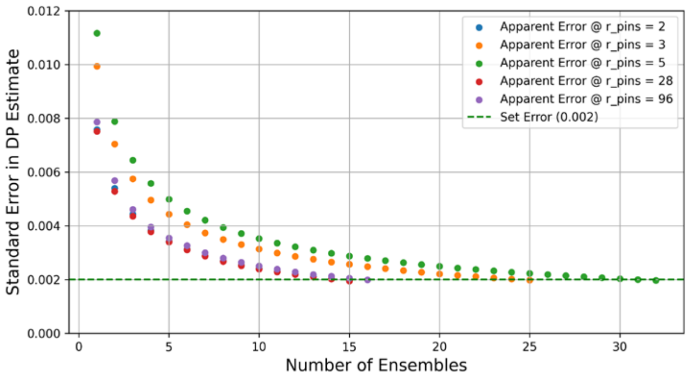

Figure 6: Convergence in standard error with number of ensembles for different r_{pin} examples.

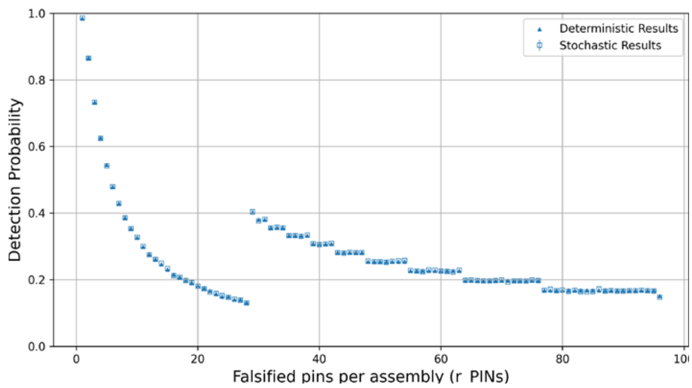

Figure 7: Co-plot of Deterministic & Stochastic Detection Probabilities for Varying r_{pin} Example.

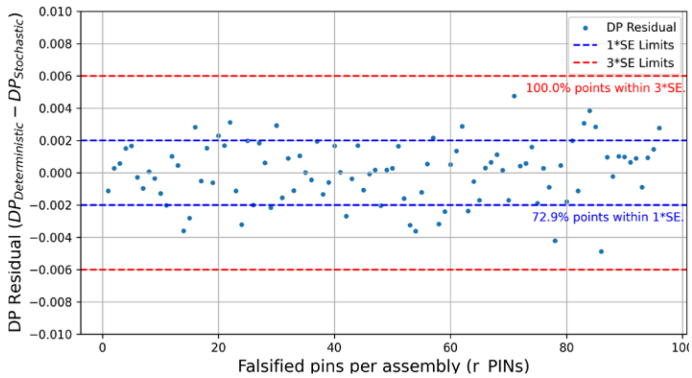

Figure 8: DP Residual plot for Varying r_{pin} example; all residuals lie within limits of three times the set error.

3.2 Example: Multi-Group Diversion

In this example, the spent fuel pond contains 2000 (*N*) Spent Fuel Assemblies with each assembly containing 96 (*L*) fuel pins, and in terms of material, each assembly contains 2 kgs or 0.25 SQ ($\bar{x}$) of Pu. A total goal amount (*G*) of 1 SQ or 8 Kg of Pu is chosen to be diverted by removing 4 pins from 21 SFAs and 30 pins from 10 SFAs while the remaining $N - 31$ assemblies remain untouched.

	Group1	Group2	Group3
Spent Fuel Assemblies	21	10	1969
Falsified Pins per Assembly	4	30	0
Total Material Diverted	(4*21 + 30*10 + 0*1969) *0.25/96 = 1 SQ		

Table 2: Multi-Group Diversion Example Case Information

Out of 2000 SFAs, the inspector verifies n_1 SFAs with the ICVD, n_2 SFAs with the DCVD, and n_3 SFAs with the PGET, where per verified SFA only one measurement instrument is applied. The instruments are the same as in the previous example. The only difference is the values of n_1, n_2, and n_3 are taken to be 59, 162 & 97 measurements, respectively. The identification probability functions, in this case, are as follows:

$$Using\ ICVD, \%Limit = 100;\ \ IP_{ICVD} = IP(r_{pins}, L, 100);\ n_1 = 59$$
$$Using\ DCVD, \%Limit = 30;\ \ IP_{DCVD} = IP(r_{pins}, L, 30);\ n_2 = 162$$
$$Using\ PGET, \%Limit = 0.38;\ \ IP_{PGET} = IP(r_{pins}, L, 0.38);\ n_3 = 97$$

The overall DP for this example given in [7] is 0.91

$$\mathrm{DP(N, n_1, n_2, n_3)} = 1 - \frac{\binom{31}{0}\binom{N-31}{n_3}}{\binom{N}{n_3}} \frac{\binom{10}{0}\binom{N-n_3-10}{n_2}}{\binom{N-n_3}{n_2}} \approx 0.9133 \tag{9}$$

The stochastic approach using the same options as the previous example produced the estimate of 0.9157 with 0.0019 as standard error, which agrees with the deterministic value of 0.9133. The residual between deterministic and stochastic DP estimates is 0.0024, which lies within limits defined by twice the stochastic standard error (± 2*SE), i.e., ± 0.0038.

4. Conclusion

The paper describes in detail the development of a stochastic approach, in section 2, to compute detection probability for inspection problems at the stratum level. In section 3, the stochastic approach is validated against two spent fuel inspection examples discussed in detail and treated deterministically in the published paper [7]. For the varying r_{pins} example, the computed DP residuals (difference between deterministic and stochastic DPs) of all the points lie within 3*SE limits depicted in Figure (8). For the multi-group diversion example, the DP residual is 0.0024, which is within 2*SE limits. Thus, the stochastic results agree with the deterministic results. The main advantage of the stochastic model over the deterministic models is its universal applicability to any inspection scenario at the stratum level (involves multi-defect stratum and multi-instrument scenarios), and the methodology remains the same making it a versatile tool. Currently, the stochastic approach can compute DPs at the stratum level. Future work involves extending the approach to compute facility-level DP and then to the state-level DP concept.

5. Acknowledgements

This work was supported by the U.S. Department of Energy/National Nuclear Security Agency/Office of International Nuclear Safeguards (NA-241), BNL-222131-2021-FORE and grant ID 357998. The authors would like to acknowledge Chris Gazze from the IAEA on numerous discussions with regards to IAEA safeguards inspections. The insights and discussions with Robert Binner and Claude Norman from the department of safeguards at the IAEA have been invaluable in the development of this work. Support and technical oversight provided by Jose Gomera and Katherine Bachner from Brookhaven National Laboratory has facilitated smooth progress of the research presented in this paper. Discussions, review and feedback from Dr. Thomas Krieger from Forschungszentrum Jülich Germany has impacted the work presented in this paper. Finally, the authors would also like to acknowledge the contributions of Ian Bleecker, Ahmed Nofal, James Porcello students from UML to this research.

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