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Editorial

Scientific Paper

A word from the Editor

With this Bulletin, number 34, ESARDA begins what is meant to become a series of “Special Issues”, addressing topical subjects or detailed studies. The idea of bringing together in a single Bulletin issue, articles on the same subject is not new. This is current practice for many journals of a scientific nature or for general public. Is ESARDA up to such a challenge? In the safeguards and non proliferation fields, is there enough scientific and technical material available to publish, once a year or every two years, a special issue totally dedicated to a given subject? You have the answer, the success will depend upon your input.

We are pleased to commence this intended series by presenting the final report of the NDA Working Group on a Multiplicity Benchmark Exercise, to test the laboratory’s capability to simulate numerically the behaviour of NDA equipment. This major piece of work undertaken by the JRC Ispra in collaboration with 11 laboratories during a 3-year programme has provided an improved understanding of the models used in neutron multiplicity counting and dead time effect prediction. It has enabled NDA practitioners to test and compare their algorithms and to prepare for future list-mode acquisition systems.

To better understand this issue, one can say that, today, neutron detectors deliver logical pulse trains following detection events. These pulse trains are processed by neutron analyzers (such as shift registers) that count the events and their time correlation. However, future systems could deliver digital signals based on time-stamping of events (list-mode). In this way the data may be directly processed by the software on the PC, therefore avoiding the use of the shift register. The algorithms tested in this exercise are the seed of these future systems.

Meanwhile, those of you with an interest in Non-Destructive Analysis will certainly find this Special Issue to be a valuable reference for the future.

I hope that the Working Groups and Members of ESARDA will support this new initiative, of which I will provide further details in the next regular issue.

If you like this idea of special issue, please remember that it is your contribution that will make it happen.

The next Bulletin issue is foreseen for this autumn.

ESARDA Multiplicity Benchmark Exercise

Final report - February 2006

Paolo Peerani, Martyn Swinhoe

List of contributors to the "ESARDA Multiplicity Benchmark Exercise".

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1 Rationale

As nuclear material standards become more difficult to obtain and less representative of the material in current facilities, calculations of neutron coincidence counting rates for Safeguards detectors are increasingly important. In the past, these calculations have often relied on the use of the "point model" equations. This limits the range of application of the technique because of the assumptions used to derive these equations. One way to bypass these assumptions is to simulate the pulse train produced by a neutron detector and analyse the pulses with "multiplicity electronics" in software. In addition, this method allows dead-time models to be used to process the pulse train before it is analysed with the "multiplicity electronics."

The ESARDA NDA working group decided to launch a benchmark in order to compare the different algorithms and codes used in the simulation of neutron multiplicity counters.

The complete simulation of a neutron counter is generally performed in two steps:

- The first one consists in the simulation of the generation, transport and detection of neutrons in the sample and detector. This is generally done using Monte Carlo codes and produces as a result a pulse train file corresponding to the time sequence of detection events (list mode data).
- The second step consists in the processing of the pulse train to simulate the operation of the acquisition electronics.

Few codes are available today to perform the first step and they are not available to everybody, so most of the potential participants had not the possibility to perform the entire exercise.

In order to derive the maximum of information and at the same time to allow a large participation, the working group decided to split the exercise into two parts with two participation levels:

- Laboratories having a full simulation capability performed the entire exercise by computing the count rates starting from the basic technical specifications.
- Those who had not the appropriate tools, received a set of pulse trains produced by one of the other participants and executed only the second phase of the exercise.

The results of participants performing the entire exercise will allow us to make a comparison among the different Monte Carlo codes for the simulation of neutron multiplicity counters.

The results of those participating to the second step only will anyway help to test the available algorithms for pulse train analysis and to derive some important information about the models applied for dead-time correction.

Only four laboratories have delivered results based on a full simulation: LANL using MCNPX, JRC using MCNP-PTA, Chalmers using MCNP-PoliMi and IPPE using the AM technique. All the others participated to the second phase only. The complete list of participants and contributors is listed above.

2 Description of the exercise

The cases to be simulated represented a simple model of a well known neutron counter into which five different types of neutron sources have been inserted:

1. A pure random neutron source (count rates 10k, 100k, 1000k Hz).
2. A pure spontaneous fission source (count rates 10k, 100k, 1000k Hz).
3. Pu metal sample (mass 10 g and 1000 g).
4. Pu oxide sample (mass 10 g and 5000 g).
5. Pu oxide sample (10 g) with the addition of an (α,n) source to give various ratios of α (= ratio of (α,n)/spontaneous fission neutrons) equal to 10, 20 and 100.

This represents a total of 13 sub-cases. Detailed specifications of the cases are reported in the next section and were used as basic input data for the preparation of the Monte Carlo models used for the full exercise.

The pulse trains generated by LANL with MCNPX have been distributed to all participants for the pulse train analysis exercise /1/. The format of each pulse train file was a simple (ASCII) list of numbers representing the detection time of a neutron in a ^{3}He filled proportional counters. They were not time sorted. The number of entries in the file was typically corresponding to 20 million events, due to a limitation in the storage capability of the post-processing code. This means that the simulated measurement time is not constant.

All participants have calculated the Singles, Doubles and Triples counting rates for the 13 pulse trains using the following parameters:

- Pre-delay = 4.5 μs
- Gate width = 64 μs
- Dead-time = 0, 0.5 and 2 μs (updating and/or non-updating)

The participants have been requested to produce the uncertainties (due to counting statistics) on each counting rate. They were encouraged to produce any other parameters of interest that may help with the comparison (e.g. “die-away time”).

3 Specifications for the pulse train generation

The detector system is an Active Well Coincidence Counter (AWCC) in fast configuration (Cd liner inside the cavity) with both disks removed (cavity height 35 cm) [ref. http://www.canberra.com/pdf/Products/Systems_pdf/jcc_51.pdf].

The sources for the five cases have been modelled as described below.

3.1 Case 1 – Random neutron source

This case has been modelled as a point isotropic source placed at the centre of the AWCC cavity (17.5 cm from the bottom). The recommended energy distribution was the Geiger-vanderZwann spectrum representing an AmLi source.

Source intensity has been tuned in order to have 10k, 100k and 1000k Hz.

From a preliminary study the efficiency of AWCC to AmLi neutrons had been estimated to be 0.373, so the forecasted source intensity was expected to be approximately 26800, 268000 and 2680000 for cases 1a, 1b and 1c respectively.

3.2 Case 2 – Pure spontaneous fission source

Again the source was described as point-like and isotropic. The energy spectrum is the one from a ^{252}Cf source. The recommended spectrum description is a Watt distribution with parameters a = 1.175 and b = 1.04.

Source intensity has been tuned in order to have 10k, 100k and 1000k Hz.

Now the efficiency of AWCC to californium spontaneous fission neutrons had been estimated to be 0.310, so the forecasted source intensity was expected to be approximately 32300, 323000 and 3230000 (corresponding to fission rates of 8600, 86000 and 860000) for cases 2a, 2b and 2c respectively.

3.3 Case 3 – Pu metal sample

The sample consists of a square (H = D) cylinder of Pu metal placed at the centre of the cavity. Being a theoretical case it was not necessary to model the details of a real container.

Assuming a density of 20 g/cm^3, the radius has been derived according to the two different masses of the subcases (0.430 cm for 10 g and 1.996 cm for 1000 g).

The simplified isotopic composition was representative of weapon-grade plutonium (90% ^{239}Pu, 10% ^{240}Pu).

The source intensity has been computed according to the spontaneous fission rate of the $^{240}Pu_{eff}$ (1 and 100 grams respectively) giving approximately 1032 and 103200 neutrons per second for cases 3a and 3b.

3.4 Case 4 – Pu oxide sample

The sample consists of a square cylinder placed at the centre of the cavity, without any additional container.

The sample is a powder of PuO_2 with a density of 2 g/cm^3 and a typical isotopic composition of a reactor-grade Pu (2% ^{238}Pu, 60% ^{239}Pu, 25% ^{240}Pu, 8% ^{241}Pu and 5% ^{242}Pu). Consequently the radius (and half-height) of the cylinder corresponding to 10 and 5000 grams of powder (8.82 and 4410 grams of Pu) results to be 0.927 and 7.355 cm respectively for cases 4a and 4b.

The source intensity has been computed according to the inherent emission from (alpha,n) and SF in oxide samples with mass of 10 and 5000 g for the two subcases (see Table 1).

Table 1: Intrinsic source intensity of the oxide samples.

		Neutron yield (neut/s/g)	Case 4a	Case 4b
Oxide mass		–	10	5000
Pu mass		–	8.82	4410
(alpha, n)	Pu-238	13860	2445	1222452
	Pu-239	39.4	209	104252
	Pu-240	145.5	321	160414
	Pu-241	1.33	1	469
	Pu-242	2.11	1	465
	Total	-	2976	1488053
Spontaneous fission	Pu-238	2535	454	227148
	Pu-240	1032	2275	1137352
	Pu-242	1726	761	380482
	Total	-	3490	1745057
Alpha ratio		–	0.853	0.853
Total source (neutrons/s)		-	6466	3233110

3.5 Case 5 – Pu oxide sample plus random source

The oxide sample is the same as in subcase 4a.

The random source is an AmLi one, placed at the centre as in case 1.

The total neutron intensity contains the contributions from the inherent Pu source and the random source. The intensity of the latter has been tuned such as to give overall alpha values of 10, 20 and 100 respectively. According to the values given in Table 1, we derived an intensity of the added source of 31900, 66800 and 346000 for cases 5a, 5b and 5c respectively.

4 Theoretical estimation of results

Due to the relative simplicity of the systems, it is possible to estimate the expected results using some theoretical model. The point model /2/ was used to compute the Singles, Doubles and Triples for zero dead-time, using the following equations:

$$S = F\varepsilon M\upsilon_{s1}(1+\alpha)$$

$$D = \frac{F\varepsilon^2 f_d M^2}{2}\left[\upsilon_{s2} + (M-1)\frac{\upsilon_{s1}\upsilon_{i2}}{\upsilon_{i1}-1}(1+\alpha)\right]$$

$$T = \frac{F\varepsilon^3 f_t M^3}{6}\left[\upsilon_{s3} + (M-1)\frac{3\upsilon_{s2}\upsilon_{i2}+\upsilon_{s1}\upsilon_{i3}(1+\alpha)}{\upsilon_{i1}-1} + 3\left(\frac{M-1}{\upsilon_{i1}-1}\right)^2 \upsilon_{s1}(1+\alpha)\upsilon_{i2}^2\right]$$

The results and the values of parameters used for the calculations are reported in Table 2. Masses and neutrons rates derive from the specifications. Efficiency and multiplication have been extracted from the MCNP results. The moments of the multiplicity distributions are nuclear data; for the induced fission distribution the values refer to fission induced from fast neutrons (1 MeV), because of the selected fast configuration of the counter where the cadmium liner stops the return of thermal neutrons from the moderator to the sample. The double gate fraction was computed assuming an approximate die-away of 50 μs, the triple gate fraction was simply assumed as the square of the doubles gate fraction.

A particular trick has been applied to the calculation of cases 5. In this case there is a mixture of two sources: the intrinsic source from the Pu oxide sample and the added AmLi source. These two sources have quite different efficiencies causing a problem of choosing the correct value of this parameter to be used into the equations. As a first approximation, we

Table 2: Point model calculations for the 13 cases.

Case #	1a	1b	1c	2a	2b	2c	3a	3b	4a	4b	5a	5b	5c
Case ID	AmLi1	AmLi2	AmLi3	Cf1	Cf2	Cf3	Metal1	Metal2	Oxide1	Oxide2	Alpha1	Alpha2	Alpha3
Mass				1.40E-08	1.40E-07	1.40E-06	10	1000	10	5000	10	10	10
AN	26800	268000	2680000	0	0	0	0	0	2976	1488000	34901	69802	349011
SF	0	0	0	32300	323000	3230000	1032	103181	3490	1745000	3490	3490	3490
S-tot	26800	268000	2680000	32300	323000	3230000	1032	103181	6466	3233000	38391	73292	352502
F	0	0	0	8600	86000	860000	478	47842	1615	807600	1615	1615	1615
Alpha ratio	infinite	infinite	infinite	0	0	0	0	0	0.853	0.853	10.00	20.00	100.00
Efficiency	0.3730	0.3730	0.3730	0.3095	0.3095	0.3095	0.316	0.316	0.300	0.302	0.370	0.370	0.370
Multiplication	1	1	1	1	1	1	1.076	1.579	1.014	1.161	1.014	1.014	1.014
υ_{s1}				3.757	3.757	3.757	2.156	2.156	2.156	2.156	2.156	2.156	2.156
υ_{s2}				11.962	11.962	11.962	3.825	3.825	3.825	3.825	3.825	3.825	3.825
υ_{s3}				31.812	31.812	31.812	5.336	5.336	5.336	5.336	5.336	5.336	5.336
υ_{i1}							3.009	3.009	3.009	3.009	3.009	3.009	3.009
υ_{i2}							7.411	7.411	7.411	7.411	7.411	7.411	7.411
υ_{i3}							14.593	14.593	14.593	14.593	14.593	14.593	14.593
D gate fraction				0.6598	0.6598	0.6598	0.6598	0.6598	0.6598	0.6598	0.6598	0.6598	0.6598
T gate fraction				0.4353	0.4353	0.4353	0.4353	0.4353	0.4353	0.4353	0.4353	0.4353	0.4353
Point model S	9996	99964	999640	10000	100000	1000001	350.4	51466	1960	1130690	14371	27435	131949
Point model D	0	0	0	3251	32509	325092	80.8	33109	198.2	202844	248.3	303.1	741.1
Point model T	0	0	0	588	5885	58850	14.0	29468	20.9	53110	28.0	35.8	98.0

assumed that Singles are mostly dominated by the AmLi neutrons, whereas Doubles and Triples are determined by the fissions. Therefore in the calculation of Singles we used the MCNP efficiency computed from case 5, whereas for Doubles and Triples we used the efficiency of sub-case 4a.

5 Results from full simulations

Scope of this part of the exercise is to have a comparison on the different codes available for the complete simulation of a neutron multiplicity counter. Only four laboratories have provided a result for the full exercise.

5.1 Methodology

LANL used the MCNPX code developed at Los Alamos /3/. MCNPX is a many particle transport code that was developed to extend MCNP to all particles and all energies. Among the many features that have been added, there are several that are directly concerned with modelling of neutron coincidence counters. These are:

- Spontaneous fission source.
- If the neutron source is specified to be spontaneous fission, the code looks for potential spontaneous fission nuclides in the source cell. The neutron energy and multiplicity distribution are taken from a table of 26 nuclides containing default data. This data can be overwritten by the user.
- Multiplicity distribution of induced fission.
- The neutron multiplicity resulting from induced fission events is similarly taken from a table of values.
- Tallying of coincidence capture events.
- A new tally was added that calculates the multiplicity distribution of the detected events from each history. The reduced moments of this distribution are calculated and these give the Singles, Doubles, Triples etc. counting rates. Because each history is tracked independently there is no contribution from accidental coincidences.
- Creation of pulse output files
- A file can be produced that contains the time and cell number of all detections that occur. Usually these files are created by neutron source events that occur at time zero. The files can be used to create pulse trains files by choosing an event rate and selecting the subsequent time behaviour of each fission history. The pulse train files can be analyzed to give Singles, Doubles, Triples etc. counting rates and used to investigate dead-time effects.

JRC used MCNP-PTA /4/. It uses a modified version of MCNP version 4C2, where the correct calculation of multiplicity distributions has been introduced together with the capability of an automatic generation of the intrinsic neutron source based on the material composition and the production of an interface file containing the following information: counter tube where pulse is generated, source event identifier, time elapsed between neutron generation and detection.

After the simulation of the neutron transport, the PTA post-processor is run: the first operation performed by PTA is the generation of a pulse train. Then the dead-time losses due to operation of the pulse-shaping amplifier are calculated. Also the dead-time losses due to the summing of the digital pulses (logical OR) are taken into account. The latter can be avoided by using a digital mixer (also modelled). Finally the remaining pulses are analysed by simulating the operations of the logic modules of the electronic chain. Available analyser models include Shift Register Analyser, Time Correlation Analyser, Pulse Interval Analyser and Multiplicity Shift Register.

Multiple independent pulse trains are generated from the same MCNP output using different sequences of random numbers.

IPPE used the AM technique described in /5/. This is based in simulating the neutron transport with MCNP and storing the detection event information in a pulse file. Then the originating events are distributed randomly according to a Poisson distribution and the absolute detection time of each event is obtained by adding to the generation time the transport time from the MCNP file. The pulse train so generated is then processed as described later in section 6.1.

To generate the pulse trains, Chalmers used the Monte Carlo code MCNP-PoliMi version 1.1 /8/. This is a code built on the ordinary MCNP version 4C code. MCNP-PoliMi was developed to simulate the full joint statistics of generation, transport and detections of neutrons and gammas in a fissile material together with time-correlations. The particles are correlated so that e.g. all neutrons from one fission are started at the same time. The particles, or source events, were started uniformly over the time intervals chosen. The PoliMi code gives as output, among other things, the time when particles reach the detector. These times were extracted from the output to form the pulse train. In case 4 and 5 the source includes several components. PoliMi is run once for each component and the corresponding pulse trains are merged into one pulse train. All pulse trains were then analysed by the group the same way as they analysed the LANL pulse trains. This method is further described in section 6.1.

5.2 Comparison of results

Table 3 reports for the 13 cases with zero dead time the following results:

Table 3: Results of the full simulation for zero dead-time.

Case #	1a	1b	1c	2a	2b	2c	3a	3b	4a	4b	5a	5b	5c
MCNPX+file proc. (LANL)													
Singles	10002	100016	1000125	9994	99765	996834	350.1	51212	2059	1265248	13097	26207	131080
Doubles				3230	32096	326364	80.2	34076	218.2	212000	247.7	309.6	768.9
Triples				583	5485	78078	14.0	34073	24.2	-72653	29.6	51.3	-312.2
MCNP-PTA (JRC)													
Singles	10001	99834	980500	9981	99633	978600	349.6	51284	2000	1121900	13976	27075	131550
Doubles	-0.05	-3.0	-583	3237	32105	296490	80.6	34299	218.4	196580	268.0	316.9	705.4
Triples	-0.28	-11	897	632	6030	39150	15.4	37175	26.3	45259	35.4	46.7	45.5
IPPE													
Singles	10000	100000	1000000	10000	100000	1000000	362.3	54930	2120	1318000	12900	27100	132000
Doubles				3242	32460	326600	101.4	36500	261.3	258100	251.3	317.3	672
Triples				541	4981	54670	19.52	42040	28.38	56070	31.2	66.21	850.2
MCNP-Polimi													
Singles	10042	100423	1004276	10064	100643	1006697	350.9	51704	2003	1159981	14065	27263	132854
Doubles	10	187	-4086	3312	33169	328926	81.8	34405	219.1	222500	278.7	316.2	655.4
Triples	-3	-336	-8182	608	6493	60520	14.7	33307	24.29	89311	36.91	38.12	-7.20
MCNPX moments													
Singles				9959	99588	995880	348.7	51143	2051	1173047	14217	27326	132197
Doubles				3205	32052	320522	79.0	33963	213.6	213189	260.8	312.6	727.3
Triples				574	5745	57448	13.6	33997	23.0	57692	29.6	37.0	95.9

Table 4: Comparison of zero dead-time results with theoretical values (point model).

Case #	1a	1b	1c	2a	2b	2c	3a	3b	4a	4b	5a	5b	5c
Singles/Point model													
LANL				0.9994	0.9976	0.9968	0.9993	0.9951	1.0507	1.1190	0.9114	0.9552	0.9934
PTA				0.9981	0.9963	0.9786	0.9978	0.9965	1.0206	0.9922	0.9725	0.9869	0.9970
IPPE				1.0000	1.0000	1.0000	1.0340	1.0673	1.0817	1.1657	0.8977	0.9878	1.0004
UNIC				1.0064	1.0064	1.0067	1.0015	1.0046	1.0219	1.0259	0.9787	0.9937	1.0069
MCNPX				0.9959	0.9959	0.9959	0.9953	0.9937	1.0467	1.0375	0.9893	0.9960	1.0019
Doubles/Point model													
LANL				0.9936	0.9873	1.0039	0.9963	1.0434	1.1021	1.0552	1.0035	1.0312	1.0571
PTA				0.9957	0.9876	0.9120	1.0013	1.0502	1.1033	0.9784	1.0860	1.0555	0.9698
IPPE				0.9973	0.9985	1.0046	1.2556	1.1024	1.3181	1.2724	1.0120	1.0470	0.9068
UNIC				1.0188	1.0203	1.0118	1.0128	1.0391	1.1054	1.0969	1.1222	1.0434	0.8844
MCNPX				0.9859	0.9859	0.9859	0.9817	1.0399	1.0788	1.0611	1.0567	1.0414	0.9999
Triples/Point model													
LANL				0.9905	0.9321	1.3267	1.0240	1.2047	1.1665	–1.4174	1.0816	1.4856	–3.3901
PTA				1.0739	1.0246	0.6653	1.1236	1.3144	1.2689	0.8829	1.2961	1.3524	0.4944
IPPE				0.9200	0.8464	0.9290	1.3959	1.4266	1.3567	1.0557	1.1115	1.8496	8.6790
UNIC				1.0324	1.1034	1.0284	1.0519	1.1303	1.1611	1.6816	1.3168	1.0650	–0.0735
MCNPX				0.9762	0.9762	0.9762	0.9946	1.2020	1.1072	1.1255	1.0841	1.0713	1.0409

- LANL results deriving from the post-processing of the pulse trains produced with MCNPX (labelled “LANL”).
- JRC results derived from pulse trains produced by MCNP-PTA (labelled “PTA”).
- LANL results deriving from the direct multiplicity tallies of MCNPX (possible only for zero dead-time cases) (labelled “MCNPX”).
- IPPE results (labelled “IPPE”).
- Chalmers results (labelled “UNIC”).

In Table 4 the results are compared with the theoretical predictions of the point model. The comparison is also illustrated in figures 1a (Singles), 1b (Doubles) and 1c (Triples).

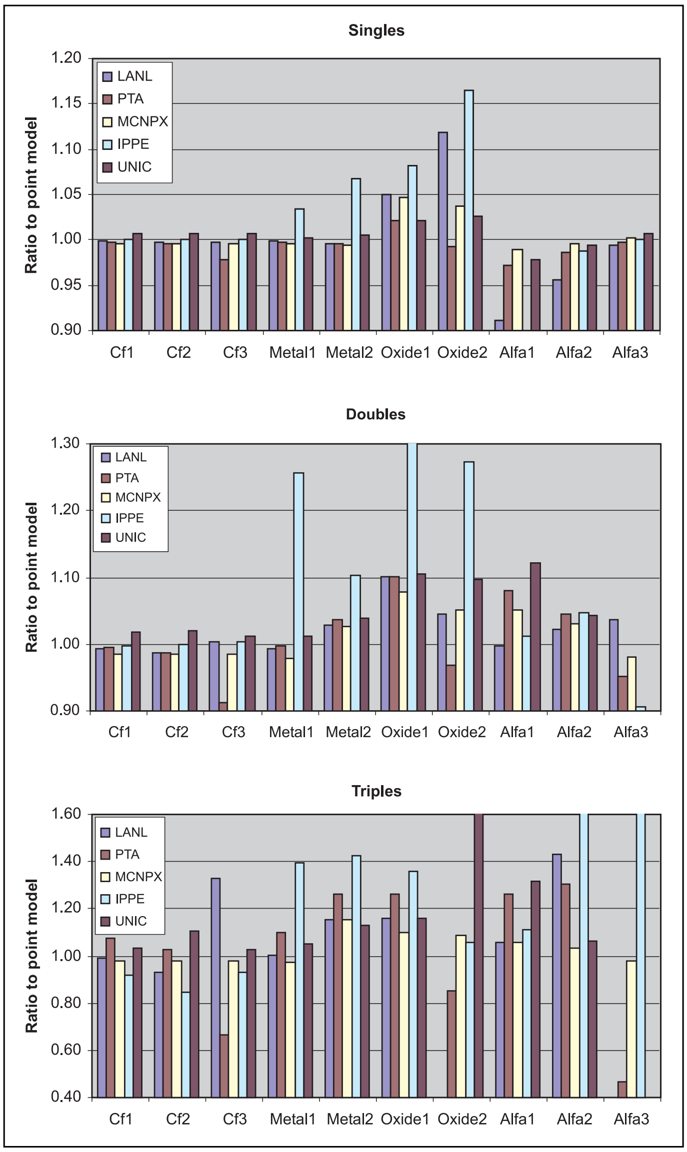

Figure 1a, 1b, 1c: Comparison of computed S, D and T to point model.

5.3 Comments on results

The results show that the direct calculation of the counting rates with MCNPX agrees reasonably well with the point model results. This shows that the point model assumptions are satisfied for this case of a well counter, although it is necessary to note the ‘trick’ mentioned in section 4, which was used to overcome the point model assumption that the detection efficiency is the same for both spontaneous fission and (α,n) neutrons.

We can compare the counting rates calculated from the “LANL”, “PTA”, “PoliMi” and “IPPE” pulse train files with each other and the point model. These results are less precise than the direct calculation because of the length of the pulse train and the inherent difficulty of extracting the true coincidence rate from the accidental coincidence rate.

We see that Singles are generally well described: there is no evidence of systematic bias and most results agree within 1 to 2% (with the only exception of IPPE results in cases 3 and 4). This is quite obvious since Singles are basically a description of neutron transport and all the methods have in common the same MCNP algorithms.

Doubles also seem to be in good agreement among the four methods. There are clear overestimations in cases 3 and 4 for IPPE.

We have to outline a known limitation in PTA explaining the deviations in cases with high count rates and zero dead-time (2c, 4b and 5c). The reason is that PTA, trying to build a realistic model, includes various components of dead-times. Some of them (amplifier dead-time, dead-time of OR_chain or derandomising buffer) are dependent on user data, some others (the length of the TTL pulse) are preset in the code. When running PTA, all the user-defined dead-time components can be set to zero, but not the pre-coded components. This means that PTA results are not exactly “zero dead-time”, but correspond to a very low dead-time (roughly 20ns). For small count rates this is negligible, but for count rates of the order of 1MHz, it may alter the results of Doubles and Triples. The effect disappears for calculations with non-zero dead-time.

In many cases the results for Triples have very large uncertainties. In particular the post processor used at LANL was limited to 20 million pulses, which only allows very short measurement periods at high rates. In two of the cases this gave negative average count rates for Triples. PTA overcomes the problem by generating a large number of different pulse trains by randomly re-distributing the time sequence of the originating events. The Chalmers group used the same measurement periods as LANL for compatibility reasons: thus they also ran into the same problems of large uncertainties, which could probably have been avoided using longer measurement times.

Despite this limitation, there is a reasonable agreement between results for Triples. PTA seems to overestimate a little bit with respect to MCNPX and to the point model as well, apart from the above problem in the cases with high count rates. This comparison between the methods of producing pulse trains gives reasonable confidence that the pulse trains produced are reliable and has no consequences for the remainder of the benchmark as only one set of pulse trains was used.

6 Results from pulse train analysis

The scope of this part of the exercise is to have a comparison on the algorithms used to process a pulse train and to model the dead-time effects. All participants have processed the same set of 13 pulse trains produced by LANL with MCNPX.

The pulse trains were simple ASCII files containing a sequence of numbers corresponding to the detection time of neutrons. MCNPX generates the file in order of generated neutron time. Since it is possible that neutrons generated at a later moment could be detected before some of the preceding ones, this resulted in a non-totally monotonically increasing order of time. The participants had to rearrange the pulse order by increasing time before analysing the files.

6.1 Methodology

The post-processor developed at LANL for the MCNPX pulse train analysis works according a procedure based on the following steps:

1. The files to be analyzed are selected.
2. The files are read into memory (Max 20M pulses).
3. The pulses are sorted into ascending time order.
4. An optional dead-time can be introduced after every pulse. This can be updating or non-updating.
5. A 'classical' shift register is implemented in which two time windows are opened for every pulse. The number of counts in each window is recorded in two histograms – one for the Reals plus Accidentals distribution, P(n), and one for the Accidentals distribution, Q(n). The first window starts at the predelay and has the width of the gate; the second window starts at the long delay and has the same width.
6. The Singles, Doubles and Triples counting rates are calculated from the histograms:

$$Singles = \sum_n P(n) / counttime$$

$$Doubles = \left[\sum_n nP(n) - \sum_n nQ(n)\right] / counttime$$

$$Triples = \left[\sum_n \frac{n(n-1)}{2}P(n) - \sum_n \frac{n(n-1)}{2}Q(n) - \frac{\sum_n nQ(n)}{\sum_n Q(n)}\left[\sum_n nP(n) - \sum_n nQ(n)\right]\right] / counttime$$

   In addition, a second value for the Doubles is calculated using the calculated accidentals.

7. A second analysis is carried out with twice the gate length in order to determine the dieaway time (assuming a single exponential dieaway).
8. Subsequently ten identical analyses are carried out each on one tenth of the pulse train in order to assess the statistical variation of the result.

The analysis performed by BNFL used two different methods:

1. Triggered R+A and A gate method
2. Triggered R+A and periodic A gate method

In method 1 two 64-μs gates were positioned ahead of the triggering pulse by intervals of 4.5 (R+A) and 1024 (A) μs. In method 2 the A-gates were calculated from an overlapping series of 64-μs gates. For the application and correction of dead-time they applied a dead-time loss function simulating a paralyzing system. The uncertainty was estimated by dividing the pulse trains in equal interval portions and measuring the mean and the variance of the resulting sub-train rates.

CEA results were produced using the AMeN code (Analyse des Mesures Neutroniques) originally developed to analyse trains from 32-channel data acquisition cards and suitably modified for this exercise in order to read the ASCII file, sort the pulses for increasing times and account for updating dead-time (in addition to the already existing non-updating one). The uncertainties were computed by dividing the pulse train in 50 sub-trains. The die-away was computed as inverse of the decay constant of the Rossi-α curve.

Chalmers University wrote a series of dedicated routines for this exercise:

1. txt2bin.c: sorting the detection events in order of time and converting the file format from ASCII to binary.
2. singles.c: calculating the Singles rate.
3. bin2dis.c: calculating the distribution of counts within a counting gate that is opened after a certain delay time. If the delay time is selected to be 4.5 μs, one obtains the R+A distribution. On the other hand, when the delay time is 4096 μs, the A distribution is obtained.
4. ud_dead.c: deleting detection events by an updating count-loss process.
5. nu_dead.c: deleting detection events by a non-updating count-loss process.
6. "bin2rad.c" calculating the die-away through the Rossi-alpha distribution

The Doubles and Triples are calculated from the Singles and the A- and A+R-distributions.

At IPPE the calculation has been done in four steps:

1. Sorting selected pulse trains.
2. Pulses removal by dead-time procedure (updating or non-updating).
3. Pulse train processing for calculation of shift register (SR) multiplicity distribution, using a common multiplicity SR scheme: Pre-delay (P=4.5 μs), Coincidence Gate (G=64 μs), Long Delay (1024 μs).
4. SR multiplicity distribution processing for count rates and σ calculation, using the SR multiplicity formalism as implemented in the INCC code /6/.

The approach used by SCK is based on the reconstruction of autocorrelation functions (Rossi-alpha method) of respectively pulse doublets and pulse triplets in the form of histograms. From these histograms the Doubles and Triples pulse rates are obtained in a simple and transparent way. The die-away time too is considered as an unknown in the analysis. The methods deliver the full count-rates. Appropriate gate fractions have to be applied when comparing with data obtained by scalers with smaller windows.

1. The totals rate λ is computed by counting all the pulses in the counting period.
2. The doubles rate is determined from the Rossi-alpha distribution of all the processed tags. The ideal Rossi-alpha distribution is given by the following equation:

   $$S_1(t) = A_1 + R_1 e^{-\frac{t}{\tau}}$$

   The reals rate Rr is obtained from the following expression:

   $$R_r = \frac{R_1 \cdot \tau}{t_m}$$

   R_1 and τ are determined from a linear regression of the logarithm of the Rossi-alpha histogram after subtraction of the mean accidentals counts $<A_1>$.
3. The Triples rate is determined from a count rate distribution called the two-dimensional Rossi alpha histogram. This histogram records triple counts in two dimensions described by an axis t_1 (time between the first and the second pulse) and an axis t_2 (time between second and the third pulse) and is described by the following function:

   $$S_2(t_1,t_2) = A_2 + C_2(e^{-\frac{t_1}{\tau}} + e^{-\frac{t_2}{\tau}} + e^{-\frac{t_1+t_2}{\tau}}) + R_2 \cdot e^{-\frac{2t_1}{\tau}}$$

   The real coincident triples count is related to the two-dimensional Rossi alpha distribution and is given by:

   $$T_r = \frac{R_2 \cdot \tau^2}{2} \cdot \frac{1}{t_m}$$

   The quantity R_2 is determined using a least squares fitting, assuming τ is known (from the one-dimensional Rossi-alpha histogram).
4. Using probability generating functions, dead-time corrections have been computed to correct the Rossi-alpha distributions for counting losses (updating counter) described by a single dead-time parameter.

CANBERRA approach to performing the multiplicity analysis of the list mode data is to subject the sorted pulse train (list of arrival times) to an ideal Shift Register Emulator created in software. Each event opens a coincidence gate of 64 µs duration after a pre-delay period of 4.5 µs. The number of events in this gate are tallied in the corresponding Reals+Accidentals, *RA(n)*, multiplicity histogram (n = 0 to 255). A second gate is opened 4096 µs after each trigger event, also of 64 µs duration, to capture the Accidentals, *A(n)*, histogram. The histograms are processed in the usual way to extract singles *(S)*, doubles *(D)*, and triples *(T)* rates. The best estimate of the mean rates is based on the treatment of the entire pulse train taking care to ensure that the last trigger event is at least 4096 µs from the last event in the train so that the final inspection of the Accidentals remains unaffected.

In order to estimate the statistical precision on the estimated rate for the entire pulse train thy calculate the standard error of the series of nominally equivalent repeat counts which are derived by splitting the pulse train into contiguous segments or cycles. The number of cycles is usually 20 or more (e.g. 20, 25 or 50 as indicated below). As an internal check for a selection of cases several choices were used. Count times are approximated from the length of the data string; i.e. between the first and last event on the pulse train. The individual cycle time is simply the total count time divided by the number of cycles. In a real measurement there may be a delay from the start until the first event and also a delay after the last neutron and the command to stop the counter. Therefore this introduces a (small) bias. The mean cycle data rates are also calculated and compared to the corresponding rates calculated from the full stream (single cycle) as a check.

They have assumed that the synthetic pulse train is an idealization in that it is for a perfect detector without dead-time. In order to generate pulse trains which are subject to dead time losses the ideal pulse trains were passed through a filter whereby events closer in time to the preceding one than a period *d* (the declared dead time of the counting chain, d = 0.5 or 2.0 µs) were removed. The modified pulse trains were then subjected to the exact same process as previously described for the ideal pulse train in order to extract dead-time perturbed estimates for the apparent *S*, *D* and *T* rates. In other words, from the ideal pulse trains dead-time affected trains were created and raw multiplicity rates (uncorrected for the dead-time previously introduced) were calculated along with associated uncertainties. Instances of both non-updating (non-paralyzable) and updating (paralyzable) dead-time were considered.

ENEA method is implemented according to the formulation given in /7/; its application procedure requires the following steps:

1. Raw data collection

   Total count rate T is recorded in time measurement t_m and each pulse triggers a set of gate lengths g_i, i = 1,2,...,K. Multiplicity distributions are obtained by collecting the number $N_{\mu,i}$ of gates of length gi with µ multiplicity (i.e. with µ pulses collected), µ =

0,1,2,…,$N_{max,I}$; $N_{max,i}$ is the maximum multiplicity observed in any gate g_i. Then the moments of the multiplicity distributions are computed:
$m_{f,i} = (T\ t_m)^{-1} \Sigma_{\mu = 1,Nmax,i}\ \mu\ N_{\mu,i}$ (first moment)
$m_{s,i} = (T\ t_m)^{-1} \Sigma_{\mu = 2,Nmax,i}\ [\mu\ (\mu-1)/2]\ N_{\mu,i}$ (second moment).

2. Data evaluation
The "effective neutron mean life time" τ is determined calculating the "Reals" rate $R_i = m_{f,i}\ T - T^2\ g_i$ (i = 1,2,...,K),
and then fitting (R_i, g_i), i = 1,2,...,K, according to the model $R = Constant*(1-e^{-g/\tau})$.
Then the correlated multiplets rates are computed according to:
$R_1 = T$
$R_{2,i} = [T\ m_{f,i} - (R_1)^2\ g_i]/f_i$
$R_{3,i} = \{T\ m_{s,i} - R_{2,i}\ R_1\ g_i\ [f + 1 - \tau/g_i\ (1-e^{-gi/\tau}] - {}^1/_2\ (R_1)^3\ g_i^2\}/f_i^2$
where f_i is the "gate fraction" = $e^{-p/\tau}\ (1-e^{-gi/\tau})$.

3. Data interpretation
From the physical interpretation of the multiplets rates a set of three equations is obtained as functions of M (multiplication factor), N_{SF} (spontaneous fission rate), N_α ((α,n) reaction rate) and ε (detection efficiency) and solved:
$R_1 = \varepsilon_i\ N_{SF,i}\ \upsilon_1\ [1 + N_{\alpha,i}/(\upsilon_1\ N_{SF,i})]$
$R_{2,i} = \varepsilon_i^2\ N_{SF,i}\ \upsilon_2$
$R_{3,i} = \varepsilon_i^3\ N_{SF,i}\ \upsilon_3$
where $\upsilon_j = \Sigma_{\upsilon=j,\upsilon max}\ \{P(\upsilon)\ \upsilon!/[(\upsilon-j)!\ j!]\}$, j=1, 2, 3.

IKI processed the pulse trains by two Delphi programs.

1. The first program.
   – Truncated the detection times to integer values.
   – Sorted the values.
   – Made an output file with time differences.

   As the detection event times was only locally disordered, only a small fraction of the data were read into memory at each step.
2. The second program processed the output of the first program in a straightforward manner.
   – Read in data for the next step in bulks of 4000 events.
   – Calculated the multiplicity for each pulse in the train and set up a time distribution for the R+A and A windows. Both windows were 64 μs wide and there was an 1024 μs between them. Pulses within the updating or non-updating dead-time were skipped.
   – Calculated the die-away time from the difference of the time distributions.
   – Calculated the Singles, Doubles and Triples rates and uncertainities from the multiplicity spectra.

The software developed at IRSN:

1. Sorts the pulse train in ascending order of time.
2. Processes the dead-time, removing unavailable pulses depending on the type (updating or non-updating) and the value of the dead-time (0.5 or 2 μs).
3. Splits the pulse train into a selected number of parts (10 sub-pulse-trains here) in order to calculate the uncertainties associated to the count rates.
4. Processes the shift register: each pulse detected opens a first gate, corresponding to the "Reals + Accidentals", 64 μs long, 4.5 μs after its detection time. Every pulses detected after this one during this range of time are counted and saved in histogram form (0 to 127). A second gate, corresponding to the Accidentals, same width, is opened 1 ms after its detection time. The distribution of the counts in this second gate is also saved in the same way. (NB: the last pulse opening the counting gates "R+A" and "A" is not the last pulse detected, but the pulse detected 2 ms before the end of the measurement, in order to count coincident pulses).
5. Calculates, for each part of the split pulse train, the measurement time, by subtracting the time of the first event that opens the counting gates from the time of the last one.
6. Calculates, for each part of the split pulse train, the single, double and triple count rates from the distributions, using the expressions given in the INCC software users manual (LA-UR-99-1291):

$$S = \frac{\sum_{i=0}^{127} (R+A)_i}{T_{measurement}} = \frac{\sum_{i=0}^{127} (A)_i}{T_{measurement}}$$

$$D = \frac{\sum_{i=1}^{127} i(R+A)_i - \sum_{i=1}^{127} i(A)_i}{T_{measurement}}$$

$$T = \frac{\sum_{i=2}^{127} \frac{i(i+1)}{2}\left[(R+A)_i - (A)_i\right] - \frac{\sum_{i=1}^{127} i(A)_i}{S} \sum_{i=1}^{127} i\left[(R+A)_i - (A)_i\right]}{T_{measurement}}$$

7. Calculates the mean value and the standard deviation of the distribution of singles, doubles and triples coming from the treatment of the 10 "sub pulse files".

6.2 Comparison of results

All the results are reported in tabular form in Appendix A and in graphical form in Appendix B (error bars refer to 1 standard deviation). Participants have been coded to keep the data presentation anonymous.

6.3 Comments of results

Generally Singles agree perfectly in all cases and in all dead-time models. Standard deviations of Singles are better than 0.03% (typically 0.01%).

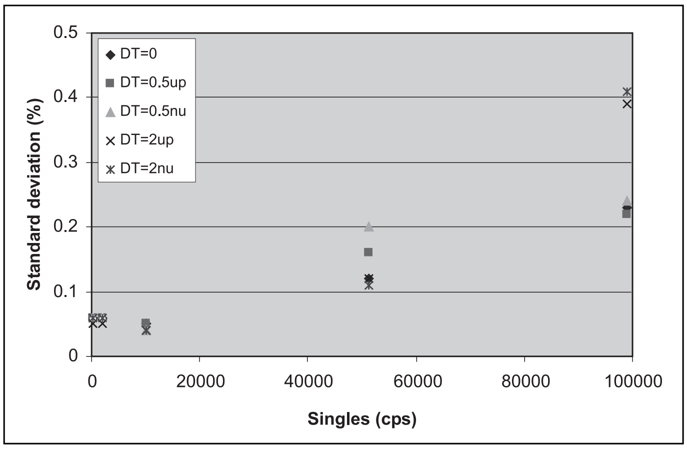

Figure 2a: Standard deviation of participant results for Doubles (cases 2-3-4).

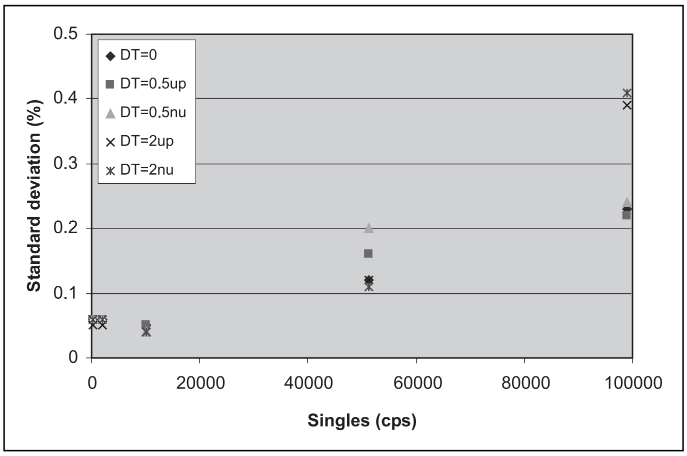

Figure 2b: Standard deviation of participant results for Triples (cases 2-3-4).

Concerning the correlated events we have generally excellent agreements when count rates are low. The results start to deviate when the Singles values are very high and/or the dead-time is large. The results of participants 2 and 3 show a systematic positive bias in the Triples. The participant 3 declared to be aware of this fact already after several comparisons with experimental pulse trains.

Non-updating dead-time appears to be easier to model, since results are less dispersed than in the updating cases.

Case 1 deals with a pure random source. Therefore only Singles have a physical meaning, Doubles and Triples are theoretically zero and non-zero results are due only to the statistical fluctuations of measurements in the (R+A) and A gates.

Cases 2, 3 and 4 show all a consistent behaviour.

With low count rates (up to 10000 cps in sub-cases 2a, 3a and 4a) the agreement is excellent, typically with a standard deviation of 0.05% for Doubles and 0.1÷0.4% for Triples. At intermediate count rates results agree within a 0.2% in Doubles, whereas Triples are still consistent in sub-case 3b (50 kHz, σ = 0.3÷0.7%), but start to diverge in sub-case 2b (100 kHz). At the very high count rates of sub-cases 2c and 4b ($\geq$1 MHz), the dispersion of results in Doubles grows to the order of percents, whereas Triples are totally out of control.

The standard deviations of participant results for these three cases are shown in figures 2a and 2b for Doubles and Triples respectively. It is evident the dependence of the standard deviation on the Singles count rate and on the dead-time.

Case 5 is a little atypical, since notwithstanding the count rates are not extremely high in absolute (13 to 130 kcps), most of the events are uncorrelated due to the high alpha ratio. This means that correlated events are subject to large uncertainties, because they result from differences between high and nearly-equal numbers. Standard deviations of results for Doubles are typically 2% for sub-cases 5a and 5b, but grow to 10÷20% for sub-case 5c. Dispersion of results for Triples ranges between 5 and 40%.

Another interesting outcome of the exercise is the verification of the dead-time correction algorithms used to extrapolate from the measured count rates the “zero dead-time” value. Some participants (BNFL, IPPE and LANL) had also provided dead-time corrected values.

The ratios of Singles, Doubles and Triples with respect to the zero dead-time values are shown in figures 3a, 3b and 3c. The results of updated dead-time values have been fitted with exponentials because it is common procedure to apply a dead-time correction of the type:

$$S/S_0 = e^{-a^*S} \qquad (1)$$

$$D/D_0 = e^{-b^*S} \qquad (2)$$

where b is commonly assumed to be equal to 4*a.

The exponential functions, which best-fit the calculated S/S_0 ratios with 0.5 and 2.0 μs updating dead-time, have parameters a equal to 0.503 and 1.999, confirming the expected results. For the Doubles the parameters b result equal to 2.52 and 9.27; these values are slightly higher than the expected values (2.0 and 8.0).

Triples ratios can also be fitted with exponentials of the same type:

$$T/T_0 = e^{-c^*S} \qquad (1)$$

but in this case it is not possible to establish a clear relation with a, since the parameters c result to be 8.63 and 72.9.

IPPE and LANL have demonstrated that reasonable dead-time corrected Triples can be obtained using the INCC algorithms /6/, with an appropriate tuning of the multiplicity dead-time parameter.

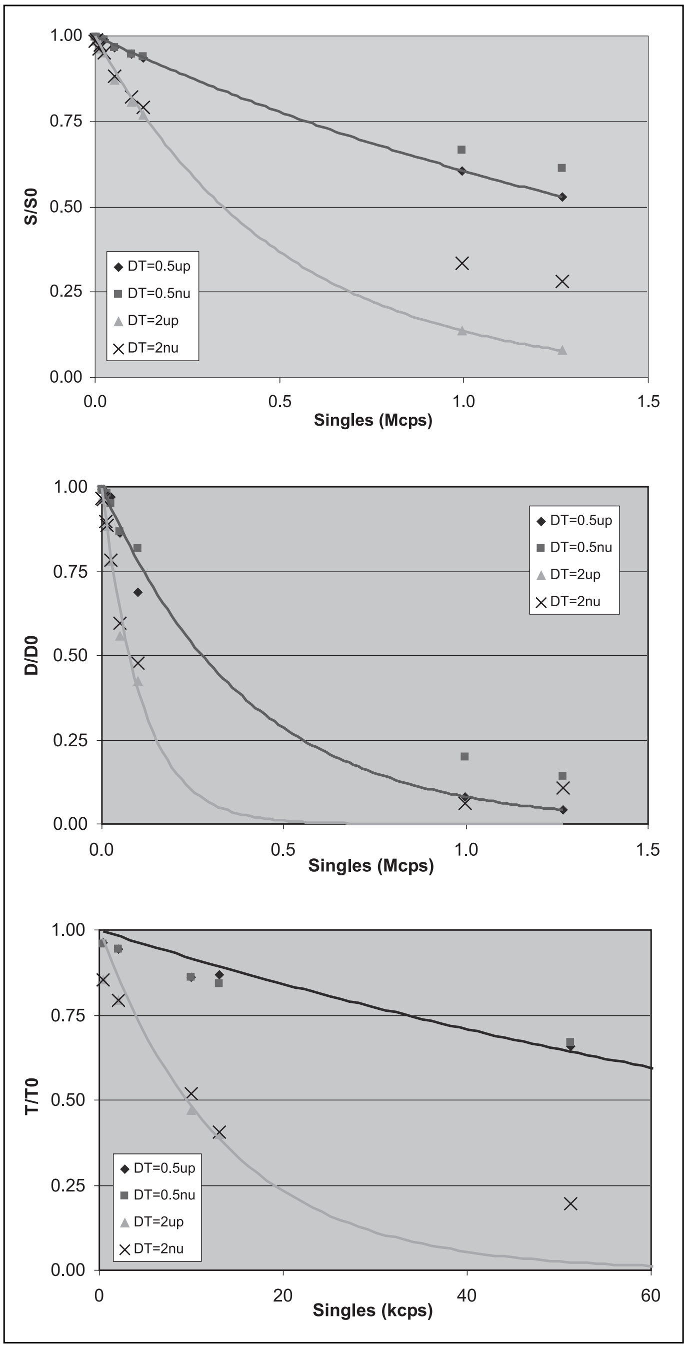

Figure 3a, 3b, 3c: Dead-time losses in Singles, Doubles and Triples.

7 Conclusions

This exercise has confirmed or revealed a number of points. Firstly, the point model, in spite of its limitations, works well for a typical well counter with "reasonable" samples. A special trick had to be used to make the point model work with the case of (α,n) neutrons having a significantly different detection efficiency from spontaneous fission neutrons. Secondly, the comparison between MCNP-PTA and MCNPX, revealed only small differences that, for the most part, are understood. This implies that they are reliable tools for the calculation of coincidence counter performance. Thirdly, the benchmark exercise itself has shown that there is a consensus among the main laboratories concerning the method of calculating Singles, Doubles and Triples from pulse trains. The results are generally well within the statistical uncertainties derived from the data, as would be expected, as all the methods process identical data. The results for the high counting rate and high (α,n) cases have large statistical uncertainties, which do not affect the intercomparison but prevent very precise conclusions being drawn about the behaviour of the counting system. With this limitation, it seems that the dead-time correction method as used in INCC works successfully on the dead-time affected rates. It is not possible to say from these results if a single stage dead-time sufficiently represents a real counter or if a multiple stage dead-time model is required that assigns dead-time to the different processes (He tube, preamplifier, Or gate). The tools that have been developed and tested can be applied to longer pulse trains for more precise investigation of dead-time effects and dead-time correction methods.

8 References

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Appendix A - Tables of the results of the pulse train analysis exercise

A.1 – Case 1

Deadtime 0µs		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 1a	10001.80	10002.49	10000.00	10001.84	10001.84	10001.83	10000.00	10001.85	10002.00	10001.77
	unc	*3.70*	*0.00*	*3.98*	*4.13*	*10.89*	*4.00*	*3.98*	*11.47*	*4.00*	*3.76*
Single rate	Case 1b	100016.10	100020.08	100000.00	100016.26	100017.16	100016.30	100000.00	100016.14	100018.00	100009.62
	unc	*38.50*	*0.05*	*39.77*	*34.41*	*111.91*	*40.01*	*39.68*	*118.10*	*39.80*	*36.40*
	Case 1c	1000118.60	1000117.79	1000000.00	1000124.60	1000140.04	1000125.00	1000000.00	1000118.59	1000125.00	999902.86
	unc	*213.70*	*0.53*	*229.40*	*213.38*	*572.68*	*200.03*	*229.30*	*601.00*	*229.00*	*273.00*
	Case 1a	-3.80	0.27	-3.13	-6.04	-2.10		-4.94	0.14	1.70	1.35
	unc	*4.80*	*0.00*	*180.00*	*5.02*	*13.52*		*4.50*	*13.92*	*4.50*	*4.81*
Double rate	Case 1b	-121.30	-6.99	-176.90	-197.08	-164.96		-169.00	4.26	102.20	39.69
	unc	*147.20*	*0.00*	*1100.00*	*155.52*	*464.44*		*141.71*	*234.45*	*142.30*	*143.91*
	Case 1c	3135.80	3975.72	-2141.00	-1557.30	-806.58		-401.18	-120.17	-3641.00	1142.25
	unc	*4892.90*	*0.03*	*19000.00*	*2255.19*	*4105.46*		*2590.00*	*5602.00*	*2586.00*	*3405.00*
	Case 1a	1.30		-17.71	-0.65	1.86		-1.17	-1.21	0.10	4.38
	unc	*2.60*		*32.00*	*2.69*	*7.85*		*10.38*	*6.79*	*11.80*	*2.17*
Triple rate	Case 1b	283.00		-2239.00	200.07	271.75		181.41	210.86	289.40	90.41
	unc	*267.80*		*3500.00*	*243.41*	*601.75*		*1410.00*	*935.00*	*2677.40*	*334.80*
	Case 1c	-147343.80		52650.00	-42251.12	-20337.80		-44700.00	-23162.59	-51738.00	-57920.61
	unc	*105582.90*		*230000.00*	*15015.71*	*22770.88*		*301000.00*	*42708.00*	*46934.00*	*21588.00*
Deadtime 0.5µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 1a	9952.00		9952.00	9952.04	9952.03	9952.04	9950.00	9952.05	9952.00	9951.98
	unc	*3.70*		*3.97*	*4.14*	*10.95*		*3.97*	*11.49*	*4.00*	*3.70*
Single rate	Case 1b	95144.00		95140.00	95143.99	95145.10	95143.90	95150.00	95143.90	95137.00	95137.63
	unc	*35.60*		*38.79*	*31.99*	*98.64*		*37.61*	*104.44*	*39.00*	*30.91*
	Case 1c	606715.70		606700.00	606792.97	606726.53	606718.00	606800.00	606714.92	606133.00	606583.34
	unc	*98.20*		*178.70*	*96.15*	*162.83*		*187.90*	*161.03*	*178.60*	*181.35*
	Case 1a	-4.00		-3.96	-5.92	-2.85		-4.97	-0.69	1.70	1.13
	unc	*4.70*		*120.00*	*4.97*	*12.97*		*4.47*	*13.25*	*4.40*	*4.58*
Double rate	Case 1b	-127.60		-159.80	-176.01	-173.41		-162.70	-55.42	72.00	45.66
	unc	*129.60*		*850.00*	*131.96*	*411.05*		*132.15*	*249.50*	*126.00*	*122.66*
	Case 1c	3164.80		644.10	1640.58	1042.51		1832.00	1115.33	-1204.00	2539.50
	unc	*1601.90*		*18000.00*	*635.76*	*1221.68*		*1940.00*	*1593.00*	*770.00*	*847.70*
	Case 1a	1.60		-17.21	-0.02	2.03		-0.70	-1.20	-0.10	4.53
	unc	*2.50*		*31.00*	*2.64*	*8.20*		*10.32*	*6.52*	*11.60*	*2.20*
Triple rate	Case 1b	225.30		-185.10	168.53	277.19		154.20	234.89	234.00	112.53
	unc	*211.70*		*2500.00*	*199.75*	*480.10*		*751.31*	*858.20*	*2254.00*	*253.60*
	Case 1c	-31829.60		-24920.00	-2471.24	-3897.94		-4720.00	-2841.29	4489.00	-3932.47
	unc	*24068.60*		*74000.00*	*1958.39*	*2369.42*		*83900.00*	*5209.00*	*85214.00*	*2610.76*
Deadtime 0.5µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 1a				9952.16	9952.15		9950.00	9952.17	9952.30	9952.10
	unc				*4.14*	*10.92*		*3.97*	*11.46*	*4.00*	*3.69*
Single rate	Case 1b				95255.70	95256.40		95300.00	95255.61	95249.00	95249.33
	unc				*32.19*	*100.19*		*38.72*	*105.70*	*39.00*	*31.12*
	Case 1c				666803.55	666846.61		667000.00	666837.18	666410.00	666692.40
	unc				*106.22*	*190.10*		*187.90*	*194.11*	*187.30*	*181.45*
	Case 1a				-5.92	-2.85		-4.97	-0.66	1.70	1.13
	unc				*4.98*	*12.99*		*4.47*	*13.25*	*4.40*	*4.58*
Double rate	Case 1b				-172.86	-173.58		-160.50	-53.55	72.40	47.29
	unc				*130.99*	*397.95*		*134.96*	*242.50*	*126.00*	*120.84*
	Case 1c				1167.76	830.58		1350.00	682.75	-1470.00	1953.61
	unc				*759.20*	*928.71*		*1740.00*	*2257.00*	*942.00*	*925.04*
	Case 1a				0.00	2.04		-0.71	-1.20	-0.10	4.49
	unc				*2.64*	*8.18*		*10.37*	*6.55*	*11.60*	*2.19*
Triple rate	Case 1b				160.33	259.80		140.50	222.67	217.00	102.55
	unc				*198.86*	*476.25*		*740.67*	*848.60*	*2263.00*	*264.25*
	Case 1c				-5038.14	-6505.85		-7910.00	-4800.32	1443.00	-10573.34
	unc				*2337.55*	*5213.73*		*107000.00*	*9474.00*	*114420.00*	*3547.83*
Deadtime 2µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 1a	9804.10		9804.00	9804.11	9804.09		9808.00	9804.11	9804.00	9804.04
	unc	*3.70*		*3.94*	*3.99*	*10.46*		*3.94*	*11.03*	*4.00*	*3.65*
Single rate	Case 1b	81893.40		81890.00	81893.64	81893.62		81900.00	81893.28	81887.00	81887.78
	unc	*29.00*		*35.99*	*28.31*	*74.51*		*35.64*	*78.67*	*36.00*	*26.45*
	Case 1c	135279.40		135300.00	135647.10	135275.91		333000.00	135279.27	135146.00	135248.64
	unc	*55.20*		*84.38*	*50.07*	*102.38*		*141.45*	*107.90*	*84.30*	*31.02*
	Case 1a	-3.60		-3.05	-5.71	-2.10		-4.28	0.52	1.60	0.89
	unc	*4.60*		*160.00*	*4.88*	*11.09*		*4.41*	*12.81*	*4.30*	*4.39*
Double rate	Case 1b	-95.60		-81.42	-112.80	-115.73		-104.30	-35.47	57.30	-19.32
	unc	*83.00*		*1300.00*	*89.81*	*310.38*		*112.81*	*283.00*	*86.90*	*88.31*
	Case 1c	456.50		182.30	934.25	141.65		638.20	685.97	180.30	221.63
	unc	*185.70*		*12000.00*	*156.63*	*396.51*		*1002.61*	*488.00*	*86.60*	*142.10*
	Case 1a	1.80		-17.36	0.80	2.94		0.10	-0.43	0.40	4.57
	unc	*2.20*		*31.00*	*2.32*	*6.92*		*10.51*	*5.28*	*11.10*	*1.77*
Triple rate	Case 1b	56.40		278.40	47.49	111.26		52.16	18.32	125.40	82.68
	unc	*117.00*		*2000.00*	*84.98*	*254.67*		*337.23*	*339.00*	*1354.00*	*105.20*
	Case 1c	-113.60		5189.00	41.41	-169.00		-115.00	-246.78	-133.70	385.98
	unc	*409.60*		*9200.00*	*251.88*	*716.84*		*8420.00*	*660.00*	*2323.00*	*304.99*
Deadtime 2µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 1a				9806.09	9806.08		9810.00	9806.09	9806.00	9806.02
	unc				*3.99*	*10.42*		*3.94*	*11.01*	*4.00*	*3.66*
Single rate	Case 1b				83343.62	83343.98		83300.00	83343.65	83339.00	83338.08
	unc				*27.74*	*78.40*		*36.23*	*82.42*	*36.00*	*26.33*
	Case 1c				333300.63	333374.51		333000.00	333372.30	333262.00	333299.41
	unc				*33.86*	*145.54*		*141.45*	*149.84*	*132.00*	*82.30*
	Case 1a				-5.65	-2.10		-4.28	0.45	1.60	0.89
	unc				*4.89*	*11.31*		*4.41*	*12.83*	*4.30*	*4.40*
Double rate	Case 1b				-118.45	-132.25		-117.75	-55.33	54.90	-4.76
	unc				*94.48*	*319.42*		*118.16*	*285.36*	*91.00*	*91.29*
	Case 1c				16004.84	15937.85		15900.00	16104.90	-20.20	15882.32
	unc				*96.99*	*241.17*		*964.61*	*199.16*	*171.00*	*133.81*
	Case 1a				0.77	2.90		0.09	-0.43	0.50	4.57
	unc				*2.32*	*6.77*		*10.34*	*5.45*	*11.10*	*1.77*
Triple rate	Case 1b				57.67	128.89		54.46	29.71	143.00	64.04
	unc				*93.18*	*314.78*		*348.23*	*388.23*	*1434.00*	*119.45*
	Case 1c				-7633.06	-7629.16		-7560.00	-7511.52	376.20	-7268.67
	unc				*156.88*	*228.19*		*14100.00*	*810.00*	*10522.00*	*279.71*

A.2 – Case 2

Deadtime 0μs		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 2a	9994.30	9994.43	9994.00	9994.25	9994.30	9994.27	9994.00	9994.26	9994.30	9994.20
	unc	*5.90*	*0.00*	*4.20*	*4.97*	*13.50*	*5.00*	*5.86*	*14.25*	*4.20*	*5.00*
Single rate	Case 2b	99764.60	99762.31	99760.00	99764.88	99765.57	99764.90	99760.00	99764.58	99762.00	99757.43
	unc	*60.00*	*0.06*	*41.88*	*50.95*	*186.68*	*49.88*	*58.42*	*198.00*	*42.00*	*56.31*
	Case 2c	996825.20	996821.14	996800.00	996833.13	996829.73	996833.80	996800.00	996825.17	996833.00	996612.01
	unc	*328.20*	*0.53*	*229.10*	*401.25*	*1156.28*	*299.05*	*321.03*	*1219.00*	*229.00*	*417.09*
	Case 2a	3234.10	3228.48	3231	3230.66	3232.18	3230.00	3234.00	3229.95	3228.30	3225.47
	unc	*7.60*	*0.00*	*19*	*6.74*	*20.78*	*8.40*	*8.62*	*29.06*	*7.50*	*9.44*
Double rate	Case 2b	32131.60	32056.39	32270.00	32137.06	32236.63	32095.70	32080.00	32090.94	32018.00	32062.02
	unc	*224.80*	*0.03*	*730.00*	*237.82*	*506.29*	*243.93*	*245.63*	*829.00*	*194.00*	*194.23*
	Case 2c	330382.30	325607.32	326500.00	326292.26	324871.43	326363.90	326800.00	326197.86	327676.00	324507.32
	unc	*6295.70*	*0.30*	*22000.00*	*5202.03*	*8144.01*	*3165.73*	*4230.00*	*7718.00*	*3160.00*	*3906.37*
	Case 2a	587.60	1329.33	687.00	580.66	582.01	582.90	584.63	582.85	576.80	568.52
	unc	*8.10*	*0.00*	*92.00*	*7.84*	*17.57*	*8.74*	*49.47*	*30.19*	*19.80*	*8.09*
Triple rate	Case 2b	5375.30	22097.46	6580.00	5163.67	5014.06	5485.40	5170.00	5475.95	5000.00	5823.27
	unc	*473.20*	*0.03*	*3800.00*	*461.11*	*1033.06*	*427.86*	*1860.00*	*1175.00*	*3845.00*	*534.51*
	Case 2c	-40950.00		-224000.00	69621.36	79002.53	78077.50	57700.00	80501.32	81972.00	74477.37
	unc	*104133.20*		*600000.00*	*25847.57*	*71114.89*	*38.60*	*255000.00*	*89726.00*	*560000.00*	*32399.87*
Deadtime 0.5μs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 2a	9897.20		9897.00	9897.20	9897.19	9897.20	9897.52	9897.19	9897.20	9897.13
	unc	*5.90*		*4.18*	*4.95*	*12.88*	*NC*	*5.81*	*13.75*	*4.20*	*4.86*
Single rate	Case 2b	94466.70		94470.00	94467.12	94467.64	94467.10	94460.00	94466.73	94459.00	94459.96
	unc	*54.50*		*40.75*	*46.98*	*177.50*	*NC*	*53.88*	*187.50*	*41.00*	*51.66*
	Case 2c	603442.30		603400.00	603520.92	603443.76	603447.90	603400.00	603441.29	602843.00	603313.08
	unc	*130.80*		*178.20*	*143.22*	*282.43*	*NC*	*204.50*	*301.62*	*178.00*	*187.84*
	Case 2a	3143.10		3140.00	3139.73	3140.71	3139.10	3142.00	3139.06	3156.00	3135.57
	unc	*7.30*		*16.00*	*6.45*	*20.26*	*NC*	*8.40*	*28.21*	*7.00*	*9.18*
Double rate	Case 2b	26031.80		26110.00	26060.14	26106.76	25975.60	26080.00	25971.99	27344.00	25982.85
	unc	*190.90*		*640.00*	*189.61*	*402.50*	*NC*	*218.10*	*652.00*	*168.00*	*169.94*
	Case 2c	31440.20		31330.00	30072.28	30106.64	30823.07	30210.00	30742.13	62042.00	30511.32
	unc	*1705.10*		*4900.00*	*1033.60*	*1818.88*	*NC*	*1744.00*	*1251.00*	*846.00*	*854.43*
	Case 2a	505.40		583.30	498.82	500.23	502.10	501.60	502.05	503.30	489.02
	unc	*7.50*		*83.00*	*7.74*	*18.30*	*NC*	*49.01*	*27.86*	*19.10*	*7.35*
Triple rate	Case 2b	57.80		3190.00	-6.83	-172.70	177.00	93.91	171.35	133.00	358.78
	unc	*360.70*		*3000.00*	*347.10*	*993.58*	*NC*	*948.40*	*763.00*	*2857.00*	*348.58*
	Case 2c	-48355.80		-37410.00	-22399.68	-24104.59	-27048.00	-28150.00	-26312.04	-43275.00	-26078.68
	unc	*23547.90*		*280000.00*	*2347.54*	*6987.48*	*NC*	*67020.00*	*7463.00*	*91273.00*	*3355.68*
Deadtime 0.5μs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 2a				9897.75	9897.72		9898.00	9897.74	9897.80	9897.68
	unc				*4.95*	*12.92*		*5.81*	*13.75*	*4.20*	*4.85*
Single rate	Case 2b				94613.67	94614.82		94610.00	94613.40	94602.60	94606.63
	unc				*47.05*	*176.30*		*54.94*	*186.50*	*40.80*	*51.70*
	Case 2c				663765.28	663798.05		663800.00	663795.21	663359.80	663653.95
	unc				*185.07*	*537.15*		*211.92*	*564.74*	*186.90*	*234.35*
	Case 2a				3140.43	3141.40		3143.00	3139.77	3156.50	3136.25
	unc				*6.46*	*20.15*		*8.48*	*28.14*	*7.30*	*9.16*
Double rate	Case 2b				26279.56	26329.63		26220.00	26189.83	27494.00	26197.56
	unc				*190.56*	*402.91*		*222.49*	*665.50*	*168.50*	*171.19*
	Case 2c				64313.58	64215.71		64400.00	64410.37	97949.00	63893.51
	unc				*1417.45*	*1809.25*		*2110.00*	*1894.00*	*1091.00*	*955.30*
	Case 2a				499.51	500.92		503.56	502.75	503.90	489.79
	unc				*7.73*	*18.13*		*49.27*	*27.81*	*19.10*	*7.32*
Triple rate	Case 2b				152.92	20.93		126.96	343.27	301.00	522.14
	unc				*342.65*	*953.27*		*1250.00*	*764.00*	*2875.00*	*342.17*
	Case 2c				-34151.05	-36888.96		-38500.00	-36869.68	-46512.00	-36534.86
	unc				*3452.02*	*11459.99*		*109000.00*	*15870.00*	*128888.00*	*5318.73*
Deadtime 2μs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 2a	9614.10		9614.00	9614.10	9614.08		9615.00	9614.10	9614.20	9614.04
	unc	*5.70*		*4.11*	*4.90*	*12.36*		*5.62*	*13.32*	*4.10*	*4.85*
Single rate	Case 2b	80339.30		80340.00	80339.82	80341.21		80340.00	80339.32	80330.70	80333.54
	unc	*41.50*		*37.58*	*36.52*	*121.21*		*46.30*	*126.54*	*37.60*	*38.85*
	Case 2c	135769.80		135800.00	136139.18	135770.34		135900.00	135769.55	135638.00	135740.19
	unc	*65.50*		*84.53*	*69.79*	*275.99*		*71.15*	*294.40*	*85.00*	*73.72*
	Case 2a	2886.80		2884.00	2883.60	2884.56		2885.00	2885.86	2948.80	2881.34
	unc	*6.50*		*17.00*	*5.89*	*20.22*		*8.03*	*26.14*	*6.80*	*8.72*
Double rate	Case 2b	13585.40		13620.00	13584.95	13618.08		13520.00	13491.98	16931.00	13504.93
	unc	*117.30*		*410.00*	*115.86*	*312.83*		*166.20*	*285.00*	*110.00*	*108.13*
	Case 2c	6087.30		5978.00	6265.25	5736.56		6331.00	5893.92	-5695.00	5758.77
	unc	*216.70*		*910.00*	*205.39*	*641.02*		*720.41*	*516.00*	*95.00*	*221.92*
	Case 2a	296.20		583.30	291.34	291.94		193.16	295.99	312.00	284.28
	unc	*5.60*		*83.00*	*5.76*	*14.62*		*29.18*	*19.61*	*17.00*	*5.95*
Triple rate	Case 2b	-4931.60		1592.00	-4880.45	-5019.95		-4962.00	-4885.61	-5624.00	-4800.54
	unc	*157.20*		*2300.00*	*149.88*	*313.54*		*471.60*	*461.00*	*1600.00*	*171.40*
	Case 2c	-3622.70		-4178.00	-3246.49	-3336.06		-3420.00	-3505.85	3198.00	-3165.67
	unc	*412.50*		*18000.00*	*326.60*	*1036.31*		*9160.00*	*974.00*	*2460.00*	*283.08*
Deadtime 2μs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 2a				9622.91	9622.90		9623.00	9622.90	9623.00	9622.85
	unc				*4.91*	*12.56*		*5.65*	*13.50*	*4.00*	*4.87*
Single rate	Case 2b				82080.31	82081.31		82080.00	82080.23	820726.00	82074.31
	unc				*38.12*	*134.77*		*47.30*	*141.24*	*38.00*	*40.80*
	Case 2c				332158.29	332229.88		332200.00	332229.18	332125.00	332158.02
	unc				*55.06*	*160.39*		*143.32*	*166.89*	*132.00*	*87.67*
	Case 2a				2893.62	2894.59		2896.00	2895.79	2955.70	2891.17
	unc				*5.91*	*20.00*		*8.10*	*26.17*	*6.90*	*8.68*
Double rate	Case 2b				15388.00	15441.28		15360.00	15288.50	18359.00	15308.40
	unc				*120.36*	*305.05*		*173.33*	*336.60*	*117.00*	*112.48*
	Case 2c				19702.37	19699.92		19730.00	19762.44	12579.00	19857.24
	unc				*173.54*	*310.22*		*984.13*	*498.00*	*188.00*	*162.42*
	Case 2a				299.72	300.33		302.80	304.24	319.00	292.70
	unc				*5.86*	*14.61*		*30.20*	*19.74*	*17.30*	*5.97*
Triple rate	Case 2b				-4458.71	-4571.91		-4490.00	-4490.12	-4953.00	-4327.00
	unc				*170.98*	*314.89*		*464.69*	*460.00*	*1733.00*	*191.79*
	Case 2c				-9049.43	-9158.04		-9240.00	-9118.80	-8899.00	-9492.77
	unc				*242.55*	*563.64*		*15800.00*	*990.00*	*11228.00*	*409.99*

A.3 – Case 3

Deadtime 0µs		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 3a	350.10	350.16	350.00	350.14	350.14	350.14	350.14	350.14	350.10	350.14
	unc	*0.20*	*0.00*	*0.23*	*0.20*	*0.56*	*0.21*	*0.19*	*0.59*	*0.10*	*0.19*
	Case 3b	51211.80	51211.82	51160.00	51211.92	51212.12	51211.70	51200.00	51211.84	51211.00	51209.62
	unc	*39.70*	*0.02*	*58.00*	*35.91*	*130.30*	*35.85*	*38.46*	*138.00*	*22.40*	*36.32*
Double rate	Case 3a	80.20	80.15	80.15	80.15	80.17	80.19	80.15	80.19	80.20	80.13
	unc	*0.10*	*0.00*	*0.41*	*0.13*	*0.21*	*0.13*	*0.11*	*0.25*	*0.10*	*0.14*
	Case 3b	34146.60	34096.36	33610.00	34148.35	34048.88	34076.30	34100.00	34078.31	34012.00	34221.87
	unc	*147.10*	*0.02*	*640.00*	*118.79*	*315.55*	*129.49*	*125.60*	*400.00*	*98.10*	*119.66*
Triple rate	Case 3a	14.00	32.14	16.23	14.03	14.05	14.05	14.04	14.05	14.10	14.00
	unc	*0.10*	*0.00*	*2.20*	*0.07*	*0.22*	*0.07*	*0.99*	*0.21*	*0.10*	*0.08*
	Case 3b	34255.00	78049.26	45770.00	34123.51	33981.54	34072.90	34100.00	34088.05	33852.00	34299.18
	unc	*434.80*	*0.03*	*16000.00*	*433.20*	*888.96*	*477.02*	*2720.00*	*1438.00*	*943.50*	*437.93*
Deadtime 0.5µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 3a	348.90		348.70	348.88	348.88	348.88	348.88	348.87	348.90	348.87
	unc	*0.20*		*0.23*	*0.20*	*0.56*	*NC*	*0.19*	*0.59*	*0.1*	*0.19*
	Case 3b	49438.30		49400.00	49438.38	49438.46	49438.20	49440.00	49438.23	49433.60	49436.11
	unc	*37.10*		*56.99*	*34.13*	*121.83*	*NC*	*36.41*	*129.00*	*22*	*33.97*
Double rate	Case 3a	79.40		79.45	79.43	79.45	79.47	79.43	79.47	79.60	79.42
	unc	*0.10*		*0.36*	*0.13*	*0.24*	*NC*	*0.11*	*0.27*	*0.1*	*0.14*
	Case 3b	29421.10		29040.00	29420.97	29333.46	29330.90	29410.00	29332.73	30664.40	29468.29
	unc	*124.70*		*580.00*	*100.53*	*275.76*	*NC*	*115.62*	*342.00*	*88.6*	*95.36*
Triple rate	Case 3a	13.50		15.93	13.47	13.48	13.48	13.48	13.48	13.60	13.44
	unc	*0.10*		*2.20*	*0.07*	*0.26*	*NC*	*0.74*	*0.24*	*0.10*	*0.08*
	Case 3b	22534.10		35400.00	22442.41	22317.67	22320.60	22480.00	22335.66	24388.00	22390.97
	unc	*307.50*		*14000.00*	*308.04*	*720.19*	*NC*	*1602.00*	*882.00*	*817.00*	*278.05*
Deadtime 0.5µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 3a				348.88	348.88		348.88	348.88	348.90	348.88
	unc				*0.20*	*0.56*		*0.19*	*0.59*	*0.10*	*0.19*
	Case 3b				49477.89	49477.85		49500.00	49477.76	49473.00	49475.64
	unc				*34.13*	*122.61*		*36.48*	*129.00*	*21.00*	*33.93*
Double rate	Case 3a				79.43	79.45		79.44	79.48	79.60	79.42
	unc				*0.13*	*0.24*		*0.11*	*0.27*	*0.10*	*0.14*
	Case 3b				29573.66	29487.06		29600.00	29490.44	30757.00	29625.70
	unc				*100.57*	*279.77*		*116.01*	*344.00*	*89.00*	*96.57*
Triple rate	Case 3a				13.47	13.48		13.48	13.48	13.60	13.44
	unc				*0.07*	*0.26*		*0.74*	*0.24*	*0.10*	*0.08*
	Case 3b				22862.64	22751.47		22900.00	22764.45	24704.00	22820.73
	unc				*309.71*	*752.77*		*1610.00*	*937.00*	*821.00*	*283.19*
Deadtime 2µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 3a	345.20		345.00	345.21	345.21		345.21	345.21	345.20	345.21
	unc	*0.20*		*0.23*	*0.19*	*0.54*		*0.19*	*0.57*	*0.10*	*0.19*
	Case 3b	44613.10		44590.00	44613.36	44613.77		44610.00	44613.12	44619.20	44611.31
	unc	*30.80*		*54.15*	*27.68*	*99.75*		*31.36*	*108.00*	*20.90*	*28.40*
Double rate	Case 3a	77.40		77.42	77.38	77.40		77.40	77.42	78.00	77.36
	unc	*0.10*		*0.30*	*0.13*	*0.21*		*0.11*	*0.26*	*0.10*	*0.13*
	Case 3b	19004.50		18860.00	18989.75	18954.85		18960.00	18953.54	22612.80	19061.77
	unc	*76.50*		*390.00*	*64.27*	*139.74*		*88.39*	*241.00*	*66.40*	*50.28*
Triple rate	Case 3a	12.00		15.00	11.96	11.96		11.96	11.96	12.20	11.92
	unc	*0.10*		*2.10*	*0.06*	*0.22*		*0.73*	*0.21*	*0.10*	*0.07*
	Case 3b	4557.00		16550.00	4481.43	4507.41		4511.00	4485.27	7630.30	4479.46
	unc	*115.70*		*8800.00*	*113.32*	*316.23*		*428.20*	*406.00*	*545.40*	*117.37*
Deadtime 2µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 3a				345.25	345.25		345.25	345.25	345.30	345.25
	unc				*0.19*	*0.54*		*0.19*	*NC*	*0.10*	*0.19*
	Case 3b				45087.05	45087.14		45100.00	45086.95	45083.00	45085.10
	unc				*28.35*	*102.00*		*31.99*	*NC*	*21.00*	*28.92*
Double rate	Case 3a				77.41	77.43		77.42	77.45	78.00	77.39
	unc				*0.13*	*0.21*		*0.11*	*NC*	*0.10*	*0.13*
	Case 3b				20328.39	20284.20		20300.00	20276.96	23537.00	20390.51
	unc				*67.71*	*159.89*		*94.76*	*NC*	*69.00*	*58.05*
Triple rate	Case 3a				11.98	11.99		11.98	11.99	12.20	11.95
	unc				*0.06*	*0.22*		*0.73*	*NC*	*0.10*	*0.07*
	Case 3b				6622.84	6644.60		6650.00	6622.75	9638.00	6630.94
	unc				*131.91*	*346.26*		*487.42*	*NC*	*572.00*	*128.02*

A.4 – Case 4

Deadtime 0µs		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 4a	2059.20	2059.24	2059.00	2059.20	2059.20	2059.20	2060.00	2059.20	2059.20	2059.20
	unc	*0.80*	*0.00*	*0.64*	*0.63*	*2.05*	*0.62*	*0.74*	*2.17*	*0.60*	*0.63*
	Case 4b	1265215.00	1265334.80	1265000.00	1265252.41	1265257.61	1265248.00	1270000.00	1265214.95	1265250.00	1264579.45
	unc	*492.30*	*1.42*	*398.00*	*486.39*	*1764.38*	*506.10*	*489.23*	*1931.00*	*398.00*	*859.29*
Double rate	Case 4a	218.00	217.96	218.00	218.00	217.88	218.20	217.92	218.20	218.20	218.40
	unc	*0.50*	*0.00*	*1.90*	*0.49*	*1.46*	*0.52*	*0.48*	*1.30*	*0.50*	*0.47*
	Case 4b	233021.00	218772.02	203000.00	214250.16	214686.71	211999.90	220000.00	211521.40	219217.00	218522.22
	unc	*17900.10*	*0.59*	*50000.00*	*7025.46*	*17949.17*	*8480.00*	*6950.00*	*25802.00*	*5659.00*	*11828.45*
Triple rate	Case 4a	24.30	55.22	25.15	24.22	24.12	24.22	24.18	24.22	24.10	24.51
	unc	*0.20*	*0.00*	*3.70*	*0.29*	*0.87*	*0.34*	*3.04*	*1.08*	*0.60*	*0.24*
	Case 4b	-467016.10		359200.00	48078.27	23231.10	-72653.00	-40500.00	-59873.33	95583.00	-58696.22
	unc	*488428.80*		*3000000.00*	*42208.15*	*120336.52*	*NC*	*799000.00*	*276777.00*	*1280000.00*	*114254.96*
Deadtime 0.5µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 4a	2053.80		2054.00	2053.80	2053.80	2053.80	2050.00	2053.80	2053.80	2053.80
	unc	*0.80*		*0.64*	*0.63*	*2.01*	*NC*	*0.73*	*2.12*	*0.60*	*0.63*
	Case 4b	670951.00		671000.00	671119.25	670956.87	670955.00	671000.00	670942.19	670109.00	670599.59
	unc	*165.10*		*289.90*	*167.29*	*615.77*	*NC*	*315.30*	*676.00*	*290.00*	*405.76*
Double rate	Case 4a	215.90		215.90	215.93	215.82	216.14	215.85	216.14	216.70	216.32
	unc	*0.50*		*2.00*	*0.49*	*1.45*	*NC*	*0.48*	*1.26*	*0.50*	*0.47*
	Case 4b	12552.50		7708.00	8780.41	8190.69	8460.00	8711.00	8510.22	23475.00	8891.26
	unc	*4281.10*		*3700.00*	*1316.54*	*2984.48*	*NC*	*1820.00*	*2900.00*	*1159.00*	*1999.38*
Triple rate	Case 4a	22.90		24.85	22.89	22.77	22.87	22.83	22.87	22.80	23.14
	unc	*0.20*		*3.70*	*0.29*	*0.80*	*NC*	*3.02*	*1.00*	*0.60*	*0.23*
	Case 4b	-83919.00		-11190.00	-5345.65	-5073.76	-18230.00	-11620.00	-15662.28	-17515.00	-15053.95
	unc	*73464.70*		*100000.00*	*3359.67*	*8973.56*	*NC*	*95300.00*	*27330.00*	*140000.00*	*11524.12*
Deadtime 0.5µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 4a				2053.81	2053.81		2050.00	2053.81	2053.80	2053.81
	unc				*0.63*	*2.01*		*0.73*	*2.12*	*0.60*	*0.63*
	Case 4b				774236.50	774309.57		774000.00	774293.41	773276.00	773899.96
	unc				*219.39*	*897.88*		*329.35*	*988.00*	*311.00*	*480.26*
Double rate	Case 4a				215.94	215.83		215.87	216.14	216.80	216.32
	unc				*0.49*	*1.45*		*0.48*	*1.26*	*0.50*	*0.47*
	Case 4b				30406.11	30229.03		31200.00	30021.89	51732.00	30813.80
	unc				*1813.15*	*3992.21*		*3400.00*	*4777.00*	*1603.00*	*3464.44*
Triple rate	Case 4a				22.90	22.78		22.85	22.88	22.80	23.15
	unc				*0.29*	*0.80*		*3.03*	*1.00*	*0.60*	*0.23*
	Case 4b				-17272.93	-17237.01		-36100.00	-36553.26	-26240.00	-39510.78
	unc				*5769.17*	*16853.57*		*240000.00*	*50165.00*	*223270.00*	*19165.46*
Deadtime 2µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 4a	2038.10		2038.00	2038.05	2038.05		2040.00	2038.05	2038.10	2038.05
	unc	*0.70*		*0.64*	*0.61*	*1.98*		*0.73*	*2.10*	*0.60*	*0.61*
	Case 4b	100983.40		101000.00	101601.40	100990.27		101000.00	100982.18	100861.00	100930.21
	unc	*103.30*		*112.50*	*98.58*	*373.57*		*142.70*	*402.00*	*112.00*	*107.38*
Double rate	Case 4a	209.90		210.00	209.96	209.85		209.91	210.15	212.60	210.36
	unc	*0.40*		*2.00*	*0.47*	*1.33*		*0.48*	*1.18*	*0.50*	*0.45*
	Case 4b	3038.50		3108.00	3871.60	3019.91		3903.00	2707.67	-1968.00	3157.80
	unc	*322.80*		*870.00*	*298.63*	*817.91*		*1624.00*	*792.00*	*91.00*	*309.67*
Triple rate	Case 4a	19.20		24.70	19.23	19.09		19.16	19.24	19.50	19.48
	unc	*0.20*		*3.70*	*0.25*	*0.66*		*3.01*	*0.86*	*0.60*	*0.19*
	Case 4b	-1137.10		-799.00	-199.04	-922.25		-10200.00	-974.37	930.00	-1549.57
	unc	*500.80*		*9800.00*	*448.49*	*988.52*		*22700.00*	*1000.00*	*1873.00*	*351.15*
Deadtime 2µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
Single rate	Case 4a				2038.23	2038.23		2040.00	2038.23	2038.20	2038.23
	unc				*0.61*	*1.98*		*0.73*	*2.11*	*0.60*	*0.61*
	Case 4b				357984.34	358101.96		358000.00	358090.99	357974.00	357909.67
	unc				*62.10*	*196.37*		*231.38*	*219.00*	*212.00*	*195.50*
Double rate	Case 4a				210.05	209.93		209.98	210.24	212.60	210.44
	unc				*0.47*	*1.34*		*0.48*	*1.18*	*0.50*	*0.45*
	Case 4b				23399.72	23430.21		23600.00	23524.03	5357.90	23539.76
	unc				*220.13*	*457.82*		*1660.00*	*879.00*	*232.90*	*415.28*
Triple rate	Case 4a				19.29	19.15		19.23	19.29	19.60	19.53
	unc				*0.25*	*0.67*		*3.02*	*0.87*	*0.60*	*0.19*
	Case 4b				-9583.80	-9333.48		-11200.00	-10472.30	-3156.00	-10828.11
	unc				*291.55*	*927.01*		*25200.00*	*3375.00*	*15222.00*	*1614.64*

A.5 – Case 5

Deadtime 0µs		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 5a	13097.40	13097.19	13100.00	13097.40	13097.31	13097.39	13100.00	13097.39	13097.20	13097.27
	unc	*5.40*	*0.00*	*5.32*	*4.77*	*8.78*	*3.93*	*5.46*	*9.22*	*5.30*	*3.79*
Single rate	Case 5b	26207.20	26207.69	26210.00	26207.18	26207.21	26207.15	26200.00	26207.16	26206.00	26206.66
	unc	*11.20*	*0.01*	*10.88*	*9.01*	*16.50*	*7.86*	*11.07*	*17.52*	*11.00*	*8.69*
	Case 5c	131079.30	131083.96	131100.00	131079.78	131078.43	131079.70	131000.00	131079.27	131080.00	131066.73
	unc	*58.30*	*0.09*	*55.37*	*51.89*	*120.50*	*52.43*	*55.79*	*128.00*	*56.00*	*43.08*
	Case 5a	254.30	260.89	264.40	257.18	260.61	247.67	252.79	247.65	259.10	247.60
	unc	*7.00*	*0.00*	*36.00*	*8.51*	*27.52*	*7.43*	*7.25*	*24.75*	*7.10*	*5.72*
Double rate	Case 5b	319.30	320.32	340.70	312.35	319.25	309.58	313.70	309.55	314.60	316.81
	unc	*21.90*	*0.00*	*160.00*	*24.39*	*54.90*	*21.98*	*20.56*	*78.76*	*20.30*	*19.87*
	Case 5c	755.10	938.30	430.50	648.40	463.50	768.88	662.24	764.23	661.70	625.15
	unc	*230.90*	*0.01*	*2000.00*	*241.55*	*873.11*	*230.66*	*229.74*	*885.00*	*227.80*	*270.67*
	Case 5a	31.50	88.07	53.35	27.47	29.65	29.58	27.35	29.58	32.60	28.60
	unc	*4.80*	*0.00*	*140.00*	*4.71*	*12.81*	*4.23*	*40.94*	*12.25*	*22.40*	*4.15*
Triple rate	Case 5b	78.90	181.50	128.50	63.85	58.38	51.31	62.85	51.23	79.40	63.42
	unc	*17.70*	*0.00*	*640.00*	*14.40*	*54.09*	*20.11*	*228.40*	*48.74*	*111.10*	*20.32*
	Case 5c	719.00		107.30	605.29	224.87	-312.19	615.88	-314.19	1082.50	226.72
	unc	*469.20*		*4800.00*	*477.80*	*1304.30*		*2390.00*	*1551.00*	*5562.70*	*491.05*
Deadtime 0.5µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 5a	13008.10		13010.00	13008.14	13008.11	13008.14	13000.00	13008.12	13008.00	13008.01
	unc	*5.30*		*5.30*	*4.84*	*8.58*	*NC*	*5.44*	*9.01*	*5.00*	*3.78*
Single rate	Case 5b	25862.30		25860.00	25862.36	26207.21	25862.30	25900.00	25862.34	25862.00	25861.84
	unc	*11.00*		*10.81*	*8.92*	*16.50*	*NC*	*11.03*	*17.26*	*11.00*	*8.72*
	Case 5c	122751.80		122800.00	122752.18	122752.80	122752.10	122800.00	122751.83	122736.90	122739.90
	unc	*52.90*		*53.59*	*49.08*	*135.79*	*NC*	*54.72*	*141.80*	*53.60*	*45.47*
	Case 5a	248.10		257.90	250.94	253.81	241.73	246.79	241.73	253.70	240.40
	unc	*6.90*		*35.00*	*8.37*	*27.67*	*NC*	*7.21*	*24.19*	*7.00*	*5.50*
Double rate	Case 5b	305.20		327.10	298.98	319.25	291.98	316.70	291.96	302.20	300.64
	unc	*21.20*		*17.00*	*23.64*	*54.90*	*NC*	*21.55*	*76.00*	*19.60*	*19.71*
	Case 5c	616.70		411.10	516.93	464.98	585.09	548.80	582.90	555.10	456.07
	unc	*192.30*		*1200.00*	*198.18*	*700.10*	*NC*	*213.73*	*614.00*	*193.40*	*220.78*
	Case 5a	26.70		49.83	23.45	25.75	26.48	23.02	26.48	28.80	24.58
	unc	*4.60*		*140.00*	*4.47*	*13.25*	*NC*	*40.91*	*12.65*	*21.90*	*3.86*
Triple rate	Case 5b	66.20		129.90	51.30	58.38	34.07	61.77	34.01	67.20	54.78
	unc	*16.60*		*680.00*	*13.31*	*54.09*	*NC*	*187.00*	*55.17*	*106.30*	*17.59*
	Case 5c	496.40		1930.00	363.84	288.02	-94.75	428.40	-95.12	1134.00	250.75
	unc	*369.00*		*4800.00*	*378.75*	*820.63*	*NC*	*1480.00*	*1283.00*	*4433.00*	*413.88*
Deadtime 0.5µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 5a				13008.45	13008.39		13000.00	13008.43	13008.00	13008.32
	unc				*4.83*	*8.57*		*5.44*	*9.00*	*5.00*	*3.78*
Single rate	Case 5b				25864.70	25864.67		25900.00	25864.67	25864.00	25864.17
	unc				*8.29*	*16.43*		*10.99*	*17.34*	*11.00*	*8.76*
	Case 5c				123002.60	123003.49		123000.00	123002.30	122989.00	122990.35
	unc				*48.74*	*131.76*		*54.00*	*137.00*	*54.00*	*44.40*
	Case 5a				250.88	253.65		246.62	241.75	254.00	240.35
	unc				*8.38*	*27.67*		*7.20*	*24.19*	*7.00*	*5.49*
Double rate	Case 5b				299.40	304.29		299.87	292.29	303.00	300.99
	unc				*23.64*	*61.73*		*20.27*	*76.50*	*20.00*	*19.73*
	Case 5c				530.43	465.23		554.89	572.68	558.00	463.23
	unc				*201.04*	*722.87*		*215.29*	*635.00*	*194.00*	*224.89*
	Case 5a				23.46	25.66		22.81	26.54	28.80	24.53
	unc				*4.46*	*13.23*		*40.88*	*12.60*	*21.90*	*3.86*
Triple rate	Case 5b				51.37	44.82		49.82	34.26	67.40	54.92
	unc				*13.22*	*49.86*		*184.20*	*55.10*	*106.30*	*17.64*
	Case 5c				368.59	334.50		436.59	-91.05	1129.00	317.00
	unc				*385.19*	*866.32*		*1600.00*	*1280.00*	*4464.00*	*419.50*
Deadtime 2µs updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 5a	12744.90		12740.00	12744.88	12744.86		12800.00	12744.87	12745.00	12744.76
	unc	*5.10*		*5.24*	*4.73*	*7.32*		*5.32*	*7.63*	*5.00*	*3.37*
Single rate	Case 5b	24853.50		24850.00	24853.53	24853.64		24900.00	24853.50	24853.00	24853.01
	unc	*10.10*		*10.59*	*8.74*	*13.61*		*10.82*	*14.57*	*11.00*	*8.10*
	Case 5c	100838.80		100800.00	100839.61	100839.58		101000.00	100838.84	100826.00	100829.34
	unc	*40.00*		*48.57*	*36.87*	*108.27*		*50.17*	*113.50*	*49.00*	*33.19*
	Case 5a	222.80		233.40	225.49	228.76		221.37	218.06	237.70	217.93
	unc	*6.40*		*35.00*	*7.66*	*26.66*		*7.01*	*21.51*	*6.60*	*5.33*
Double rate	Case 5b	246.90		276.10	243.78	250.07		244.60	238.80	266.00	245.22
	unc	*18.90*		*200.00*	*20.80*	*54.33*		*19.39*	*69.27*	*18.00*	*19.18*
	Case 5c	334.20		330.40	249.97	363.03		328.52	262.29	264.00	198.09
	unc	*116.80*		*580.00*	*118.13*	*483.72*		*169.24*	*318.00*	*120.00*	*143.35*
	Case 5a	12.60		37.17	9.96	12.54		10.03	13.23	16.40	12.96
	unc	*4.10*		*110.00*	*3.92*	*12.46*		*29.15*	*10.20*	*20.50*	*3.54*
Triple rate	Case 5b	27.30		113.10	18.44	17.23		19.97	10.41	40.70	24.34
	unc	*13.50*		*730.00*	*11.07*	*44.74*		*79.38*	*47.00*	*93.10*	*12.07*
	Case 5c	-65.20		2565.00	-43.34	-54.59		-61.30	-244.75	202.00	-96.11
	unc	*159.80*		*4100.00*	*148.33*	*303.45*		*584.71*	*436.00*	*2275.00*	*140.96*
Deadtime 2µs non updating		ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
	Case 5a				12749.81	12749.77		12800.00	12749.80	12750.00	12749.69
	unc				*4.73*	*7.42*		*5.38*	*7.66*	*5.00*	*3.37*
Single rate	Case 5b				24887.39	24887.60		24900.00	24887.36	24887.00	24886.88
	unc				*8.75*	*13.82*		*10.76*	*14.79*	*11.00*	*8.17*
	Case 5c				103838.62	103838.83		104000.00	103838.65	103828.00	103828.76
	unc				*39.11*	*109.18*		*49.50*	*114.00*	*49.00*	*32.75*
	Case 5a				226.10	229.34		222.24	218.76	238.00	218.54
	unc				*7.70*	*26.64*		*7.03*	*21.25*	*7.00*	*5.40*
Double rate	Case 5b				245.79	252.53		245.82	240.54	269.00	248.30
	unc				*21.01*	*54.43*		*19.42*	*70.44*	*18.00*	*19.09*
	Case 5c				260.12	343.63		305.35	280.56	306.00	212.45
	unc				*127.01*	*548.14*		*181.03*	*338.00*	*128.00*	*143.40*
	Case 5a				10.47	12.75		10.35	13.68	16.60	13.12
	unc				*3.95*	*12.47*		*29.17*	*10.20*	*20.50*	*3.56*
Triple rate	Case 5b				19.33	18.83		21.12	11.42	43.00	26.03
	unc				*11.12*	*42.80*		*83.40*	*46.00*	*94.00*	*12.00*
	Case 5c				32.50	26.41		-9.83	-259.62	280.00	-62.54
	unc				*163.75*	*322.08*		*684.47*	*407.00*	*2502.00*	*183.02*

Appendix B - Graphs of comparisons of the pulse train analysis exercise

B.1 – Case 1

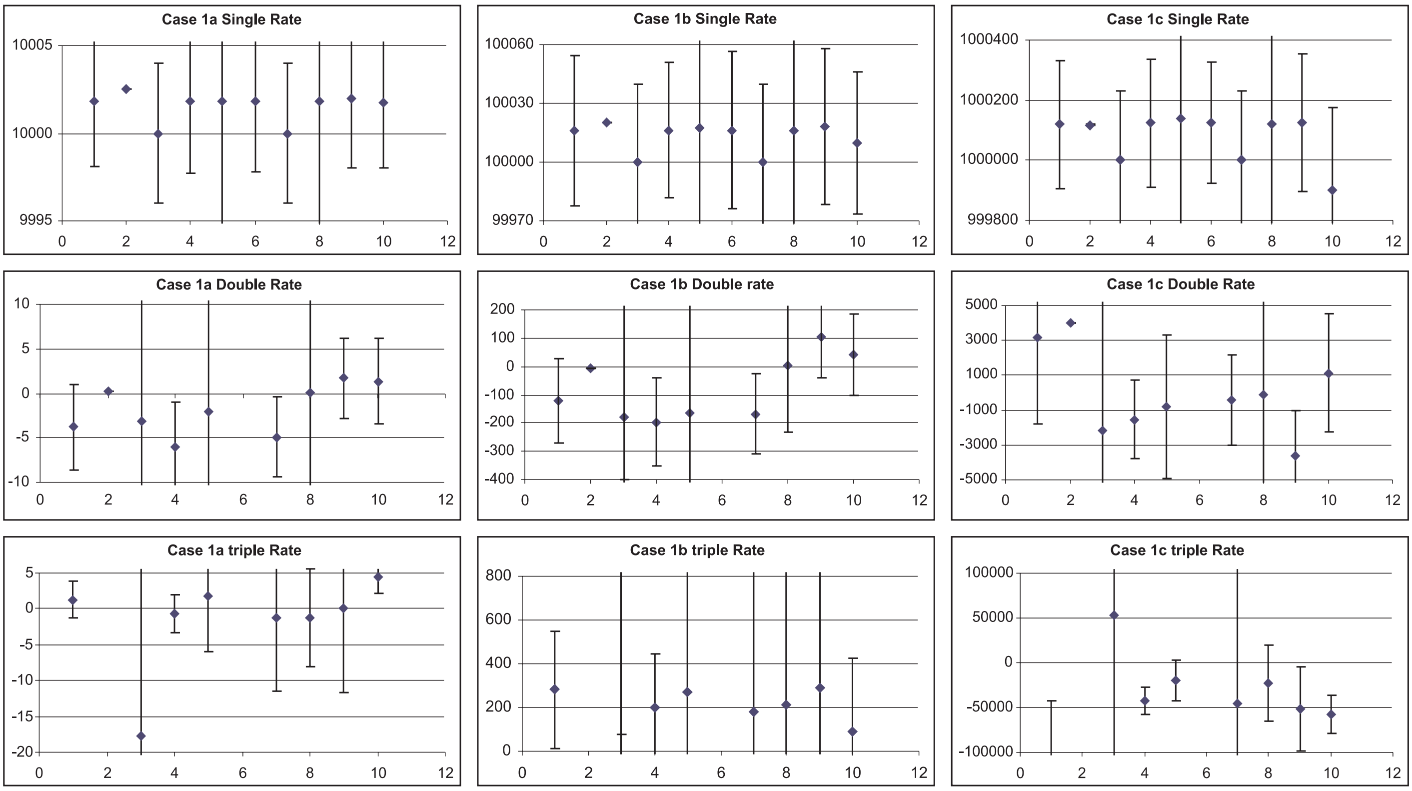

Figure B.1.1: Results for case 1 at zero dead-time.

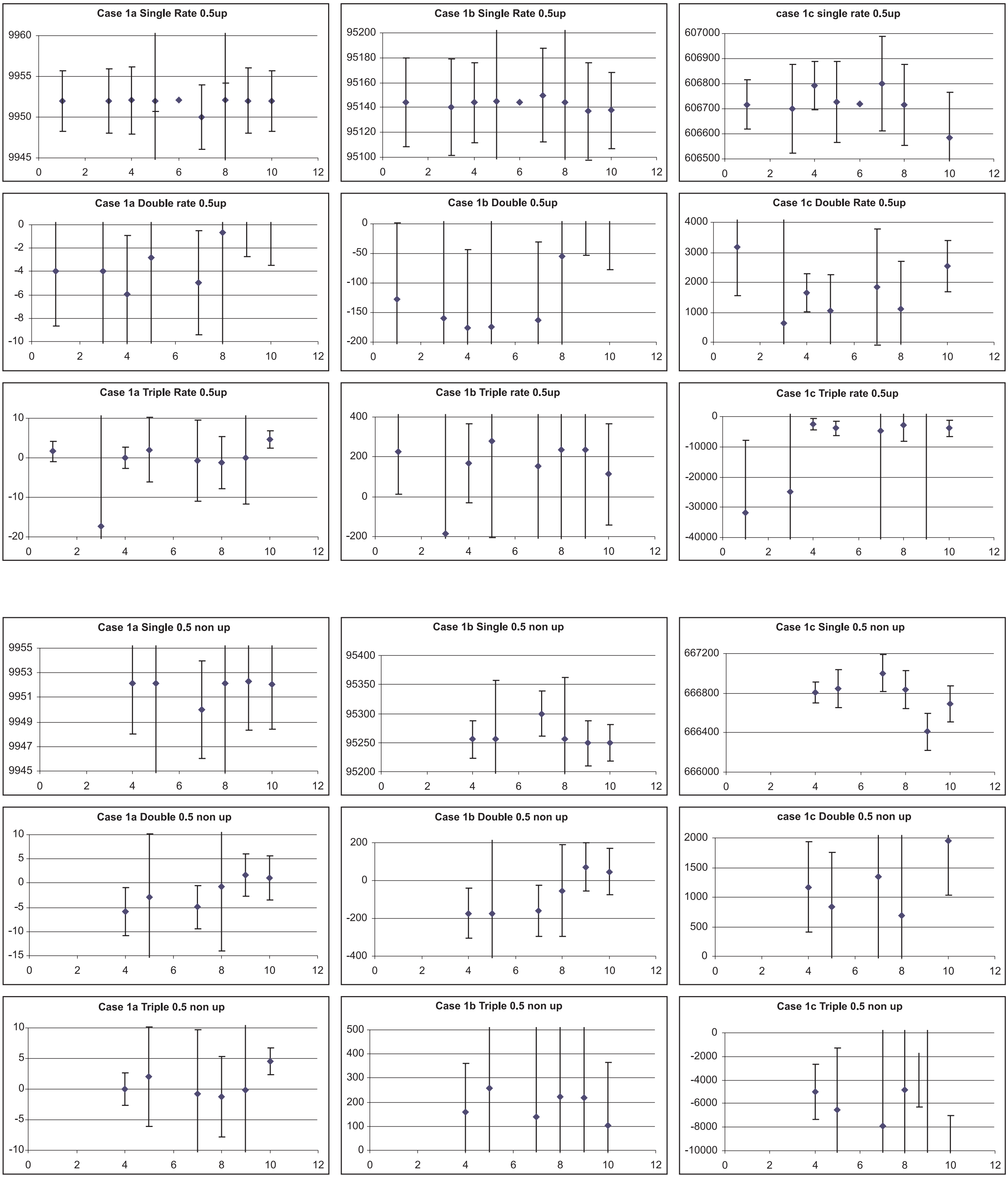

Figure B.1.2: Results for case 1 at 0.5 μs dead-time (updating and non-updating).

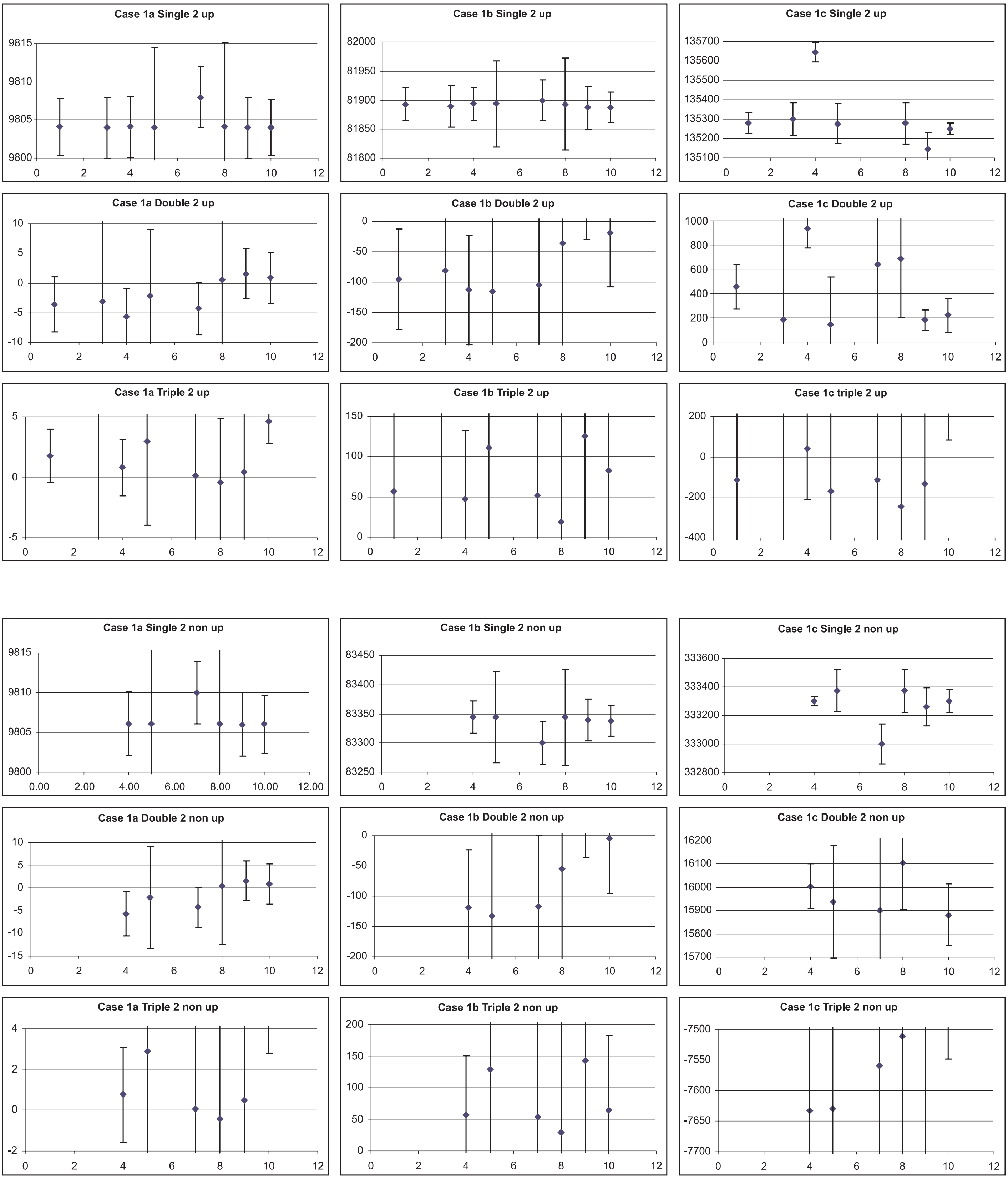

Figure B.1.3: Results for case 1 at 2 μs dead-time (updating and non-updating).

B.2 – Case 2

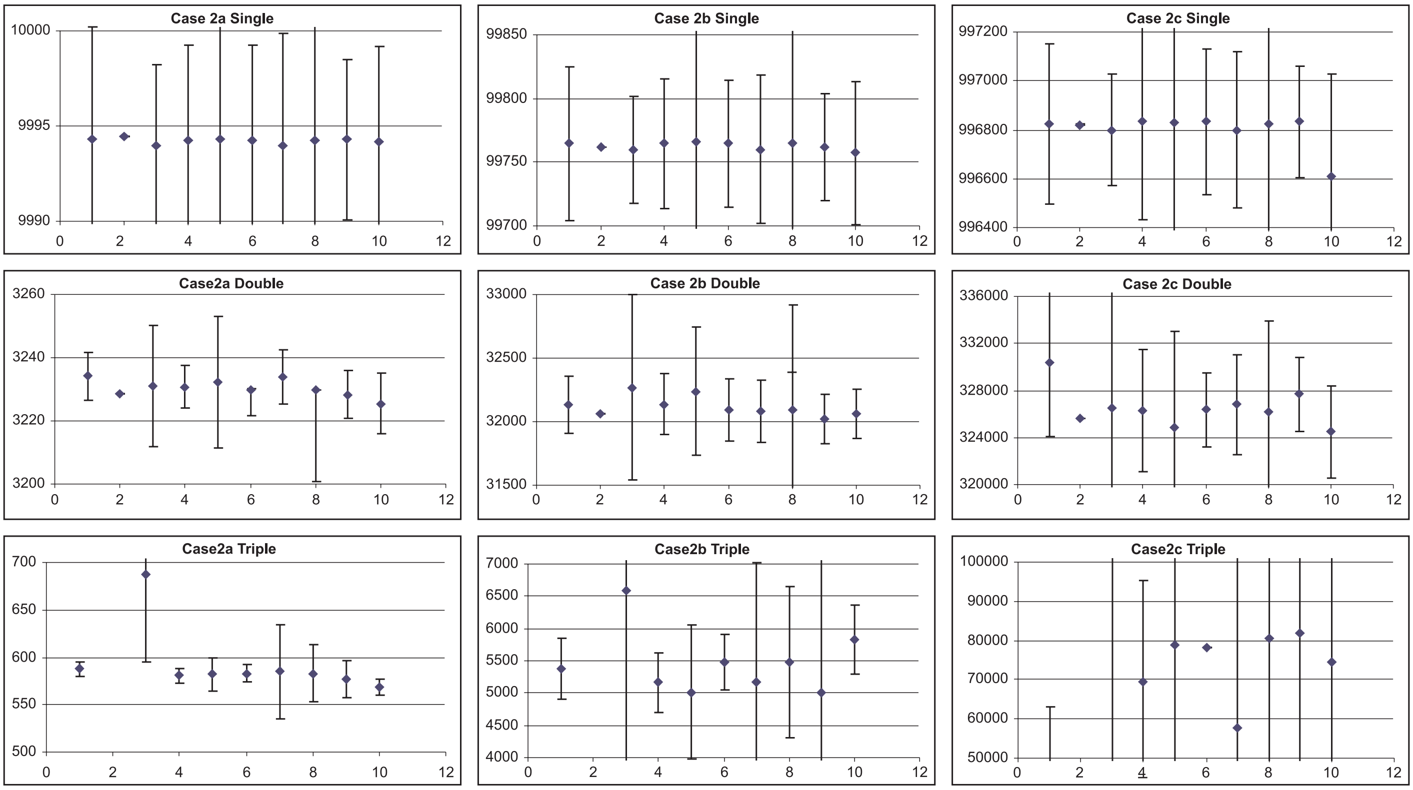

Figure B.2.1: Results for case 2 at zero dead-time.

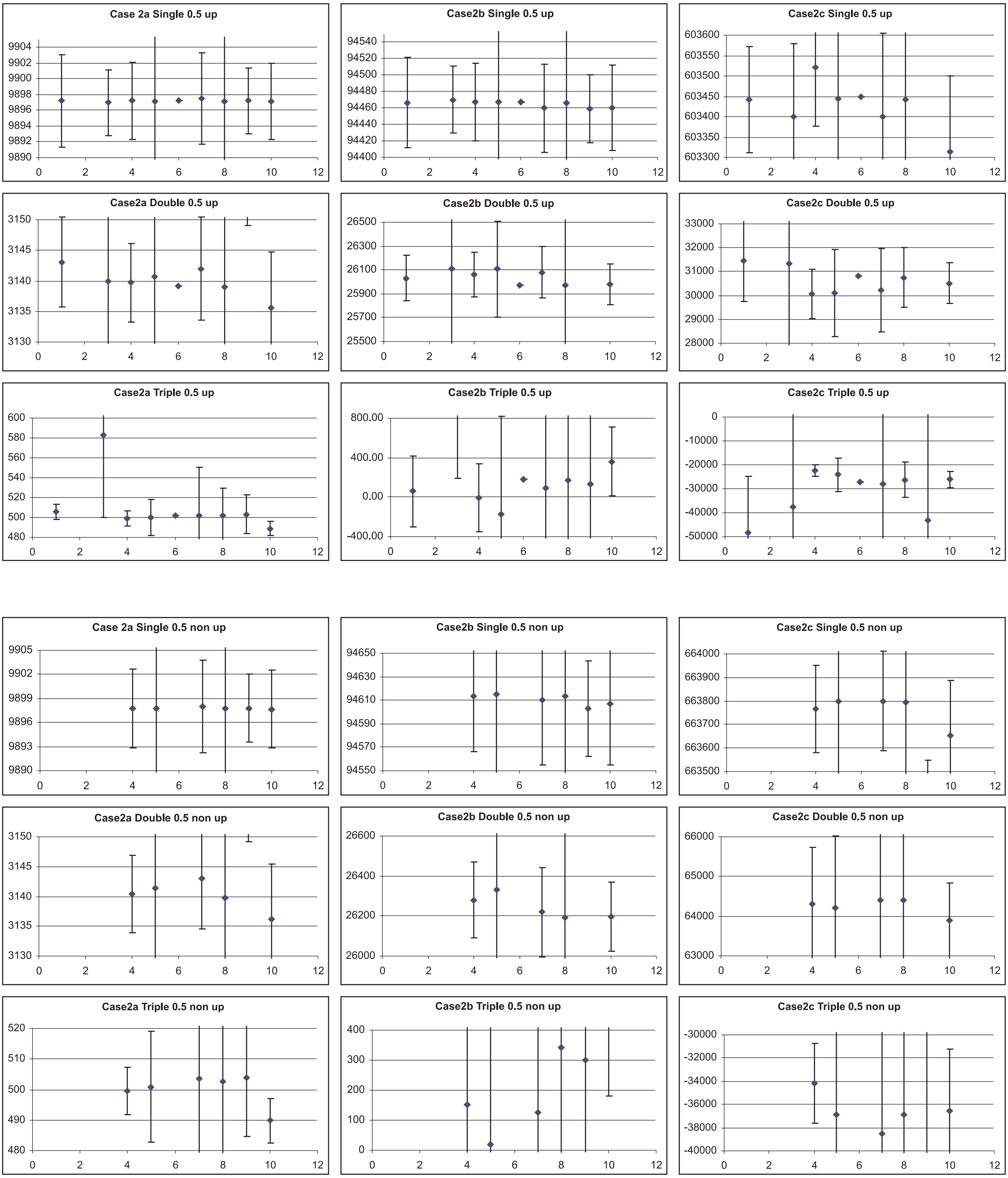

Figure B.2.2: Results for case 2 at 0.5 µs dead-time (updating and non-updating).

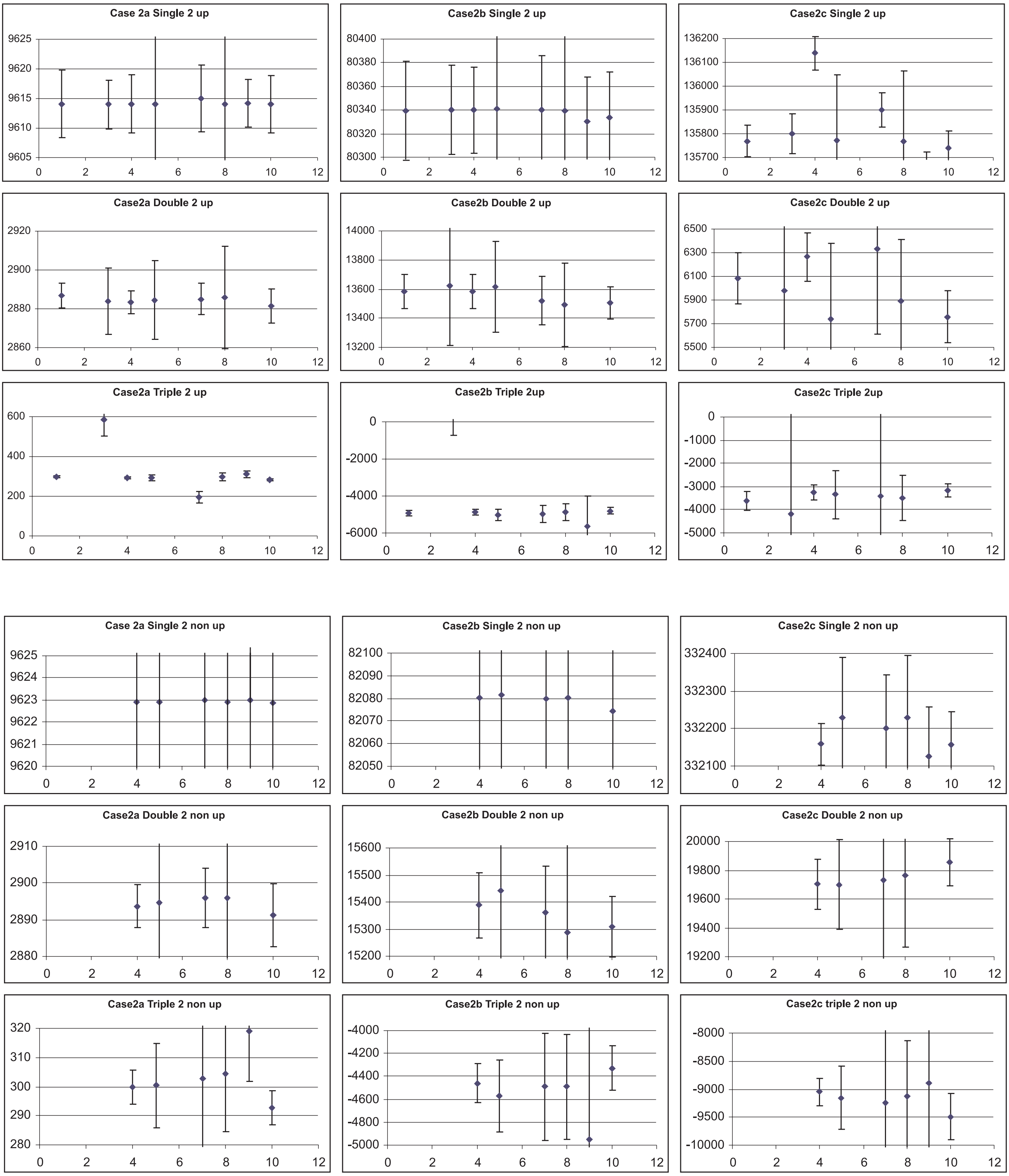

Figure B.2.3: Results for case 2 at 2 µs dead-time (updating and non-updating).

B.3 – Case 3

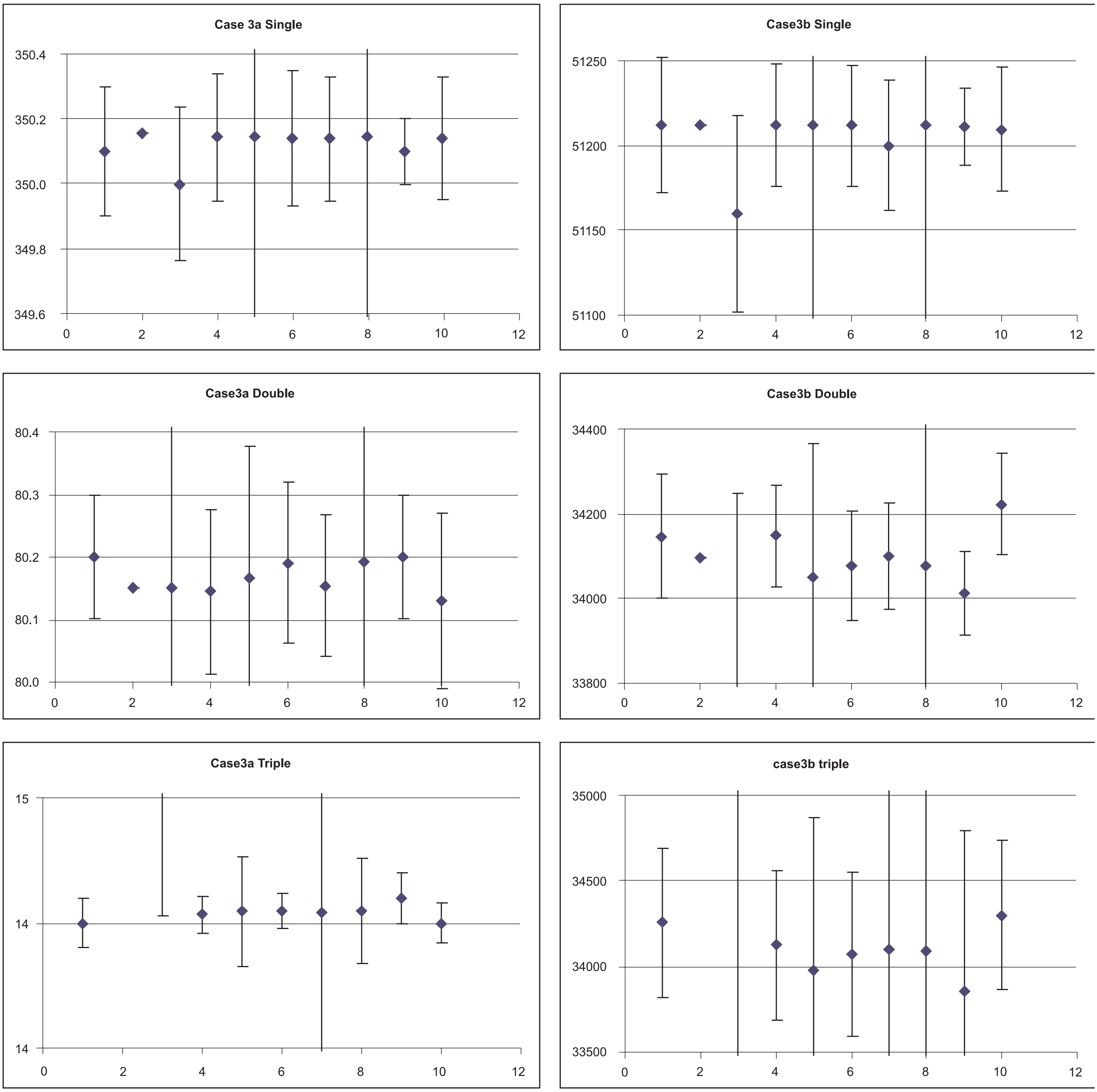

Figure B.3.1: Results for case 3 at zero dead-time.

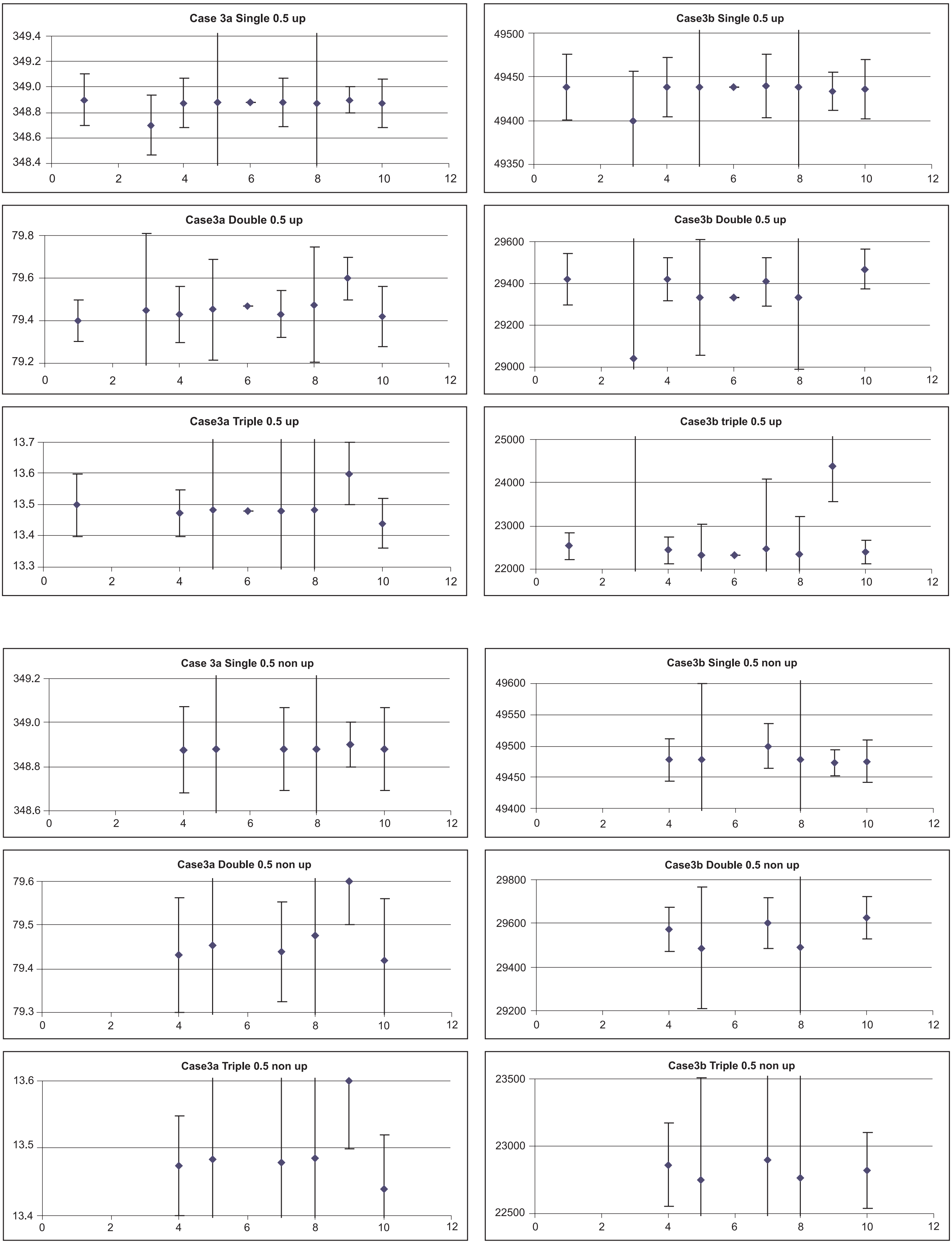

Figure B.3.2: Results for case 3 at 0.5 µs dead-time (updating and non-updating).

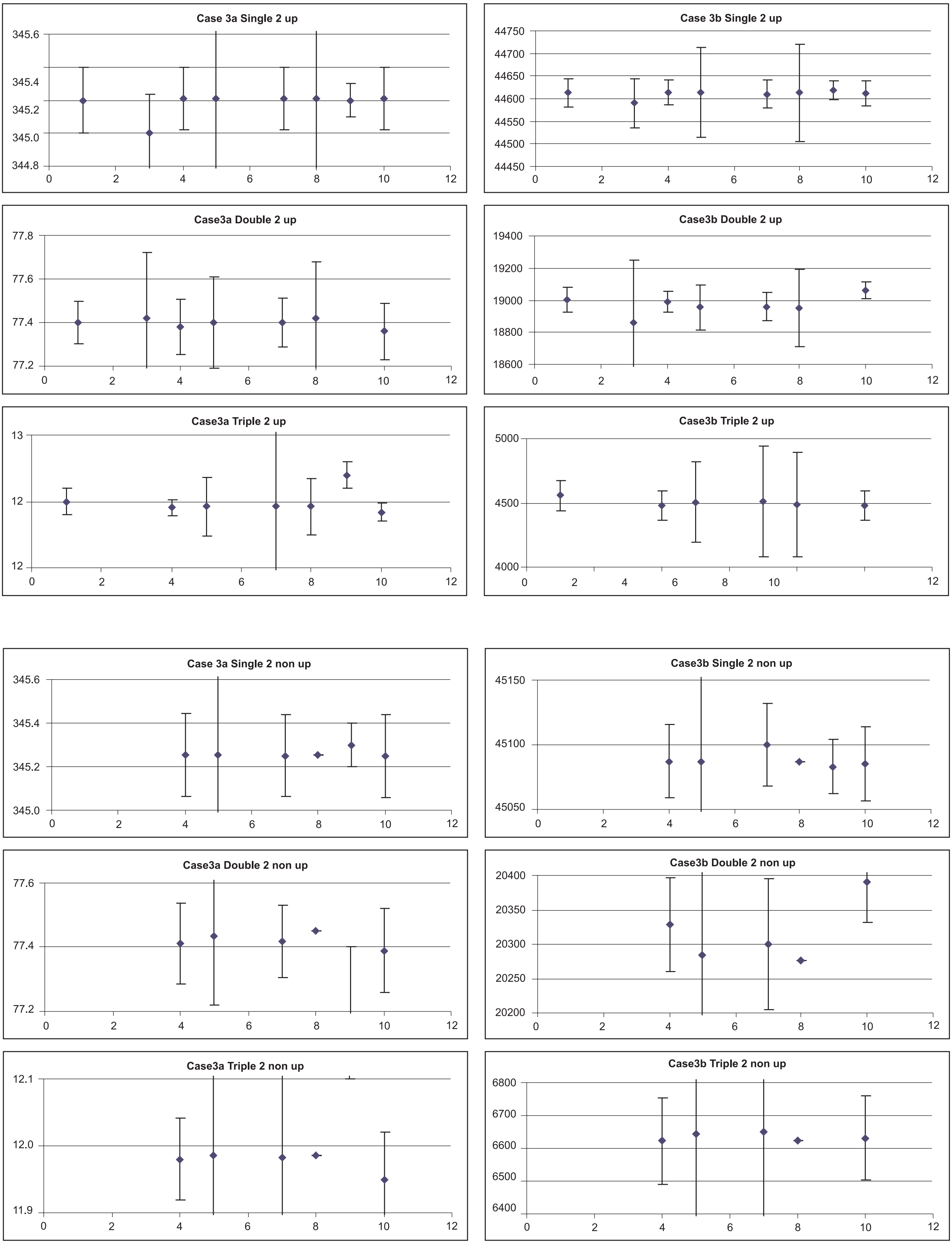

Figure B.3.3: Results for case 3 at 2 μs dead-time (updating and non-updating).

B.4 – Case 4

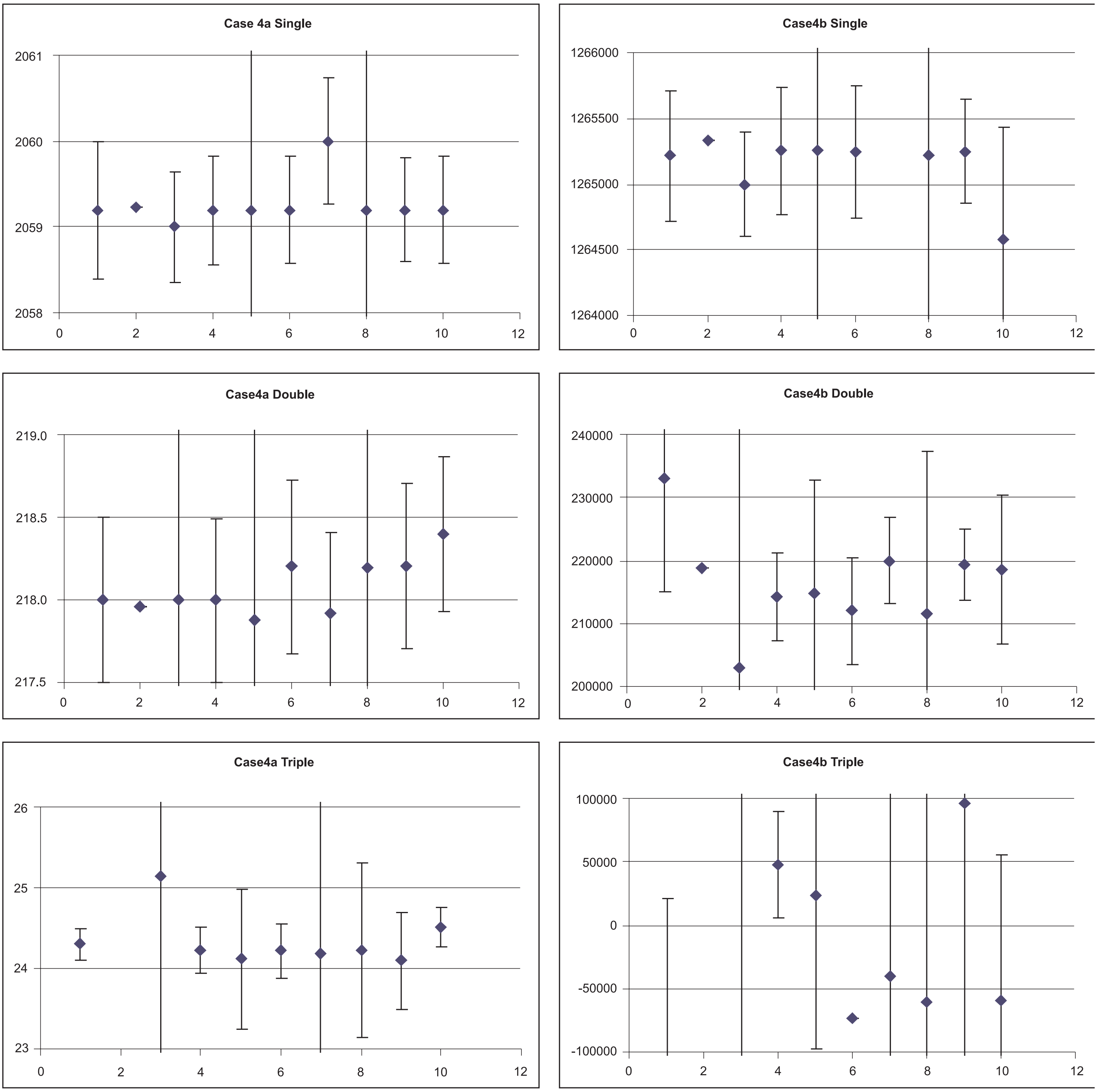

Figure B.4.1: Results for case 4 at zero dead-time.

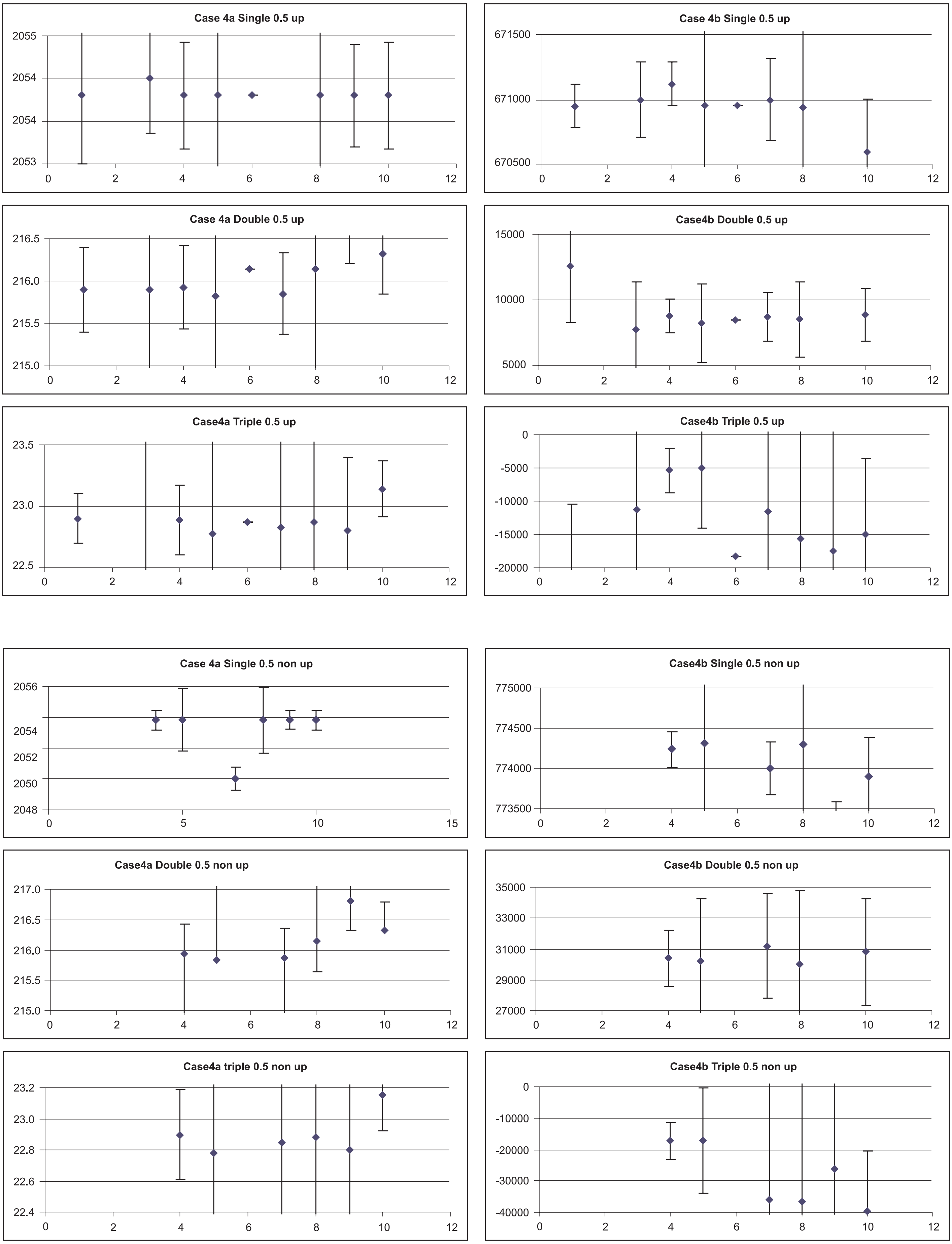

Figure B.4.2: Results for case 4 at 0.5 μs dead-time (updating and non-updating).

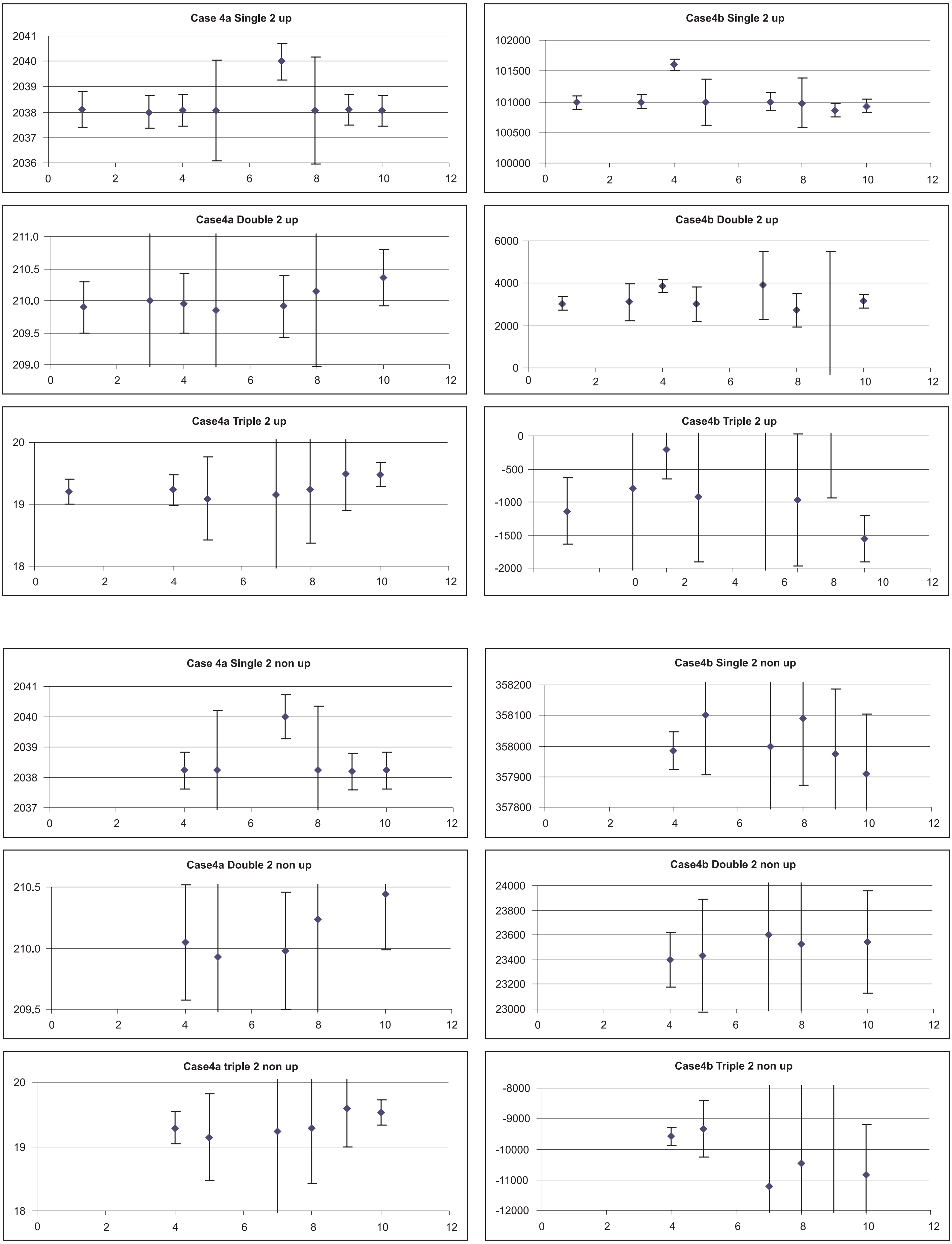

Figure B.4.3: Results for case 4 at 2 μs dead-time (updating and non-updating).

B.5 – Case 5

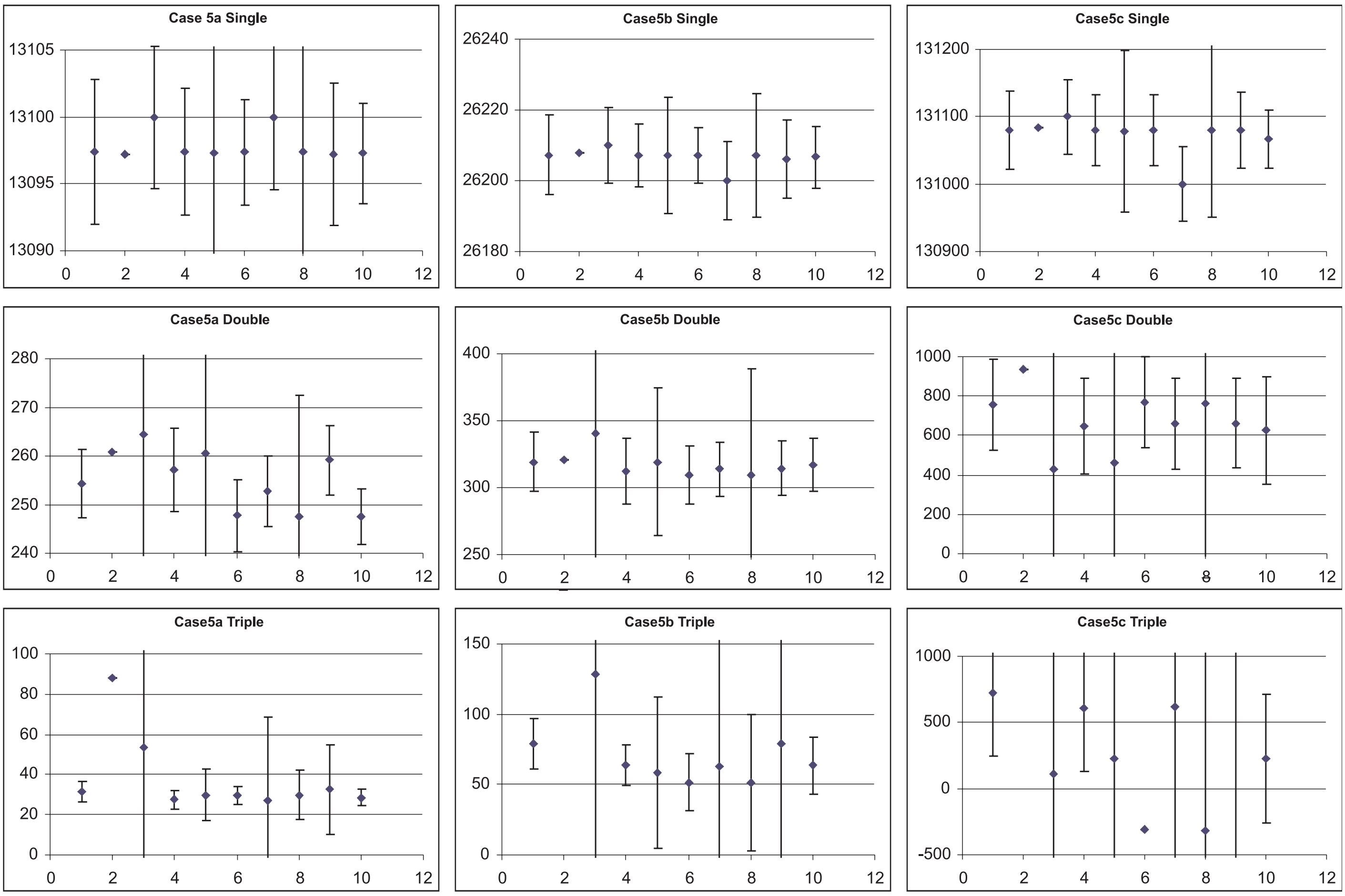

Figure B.5.1: Results for case 5 at zero dead-time.

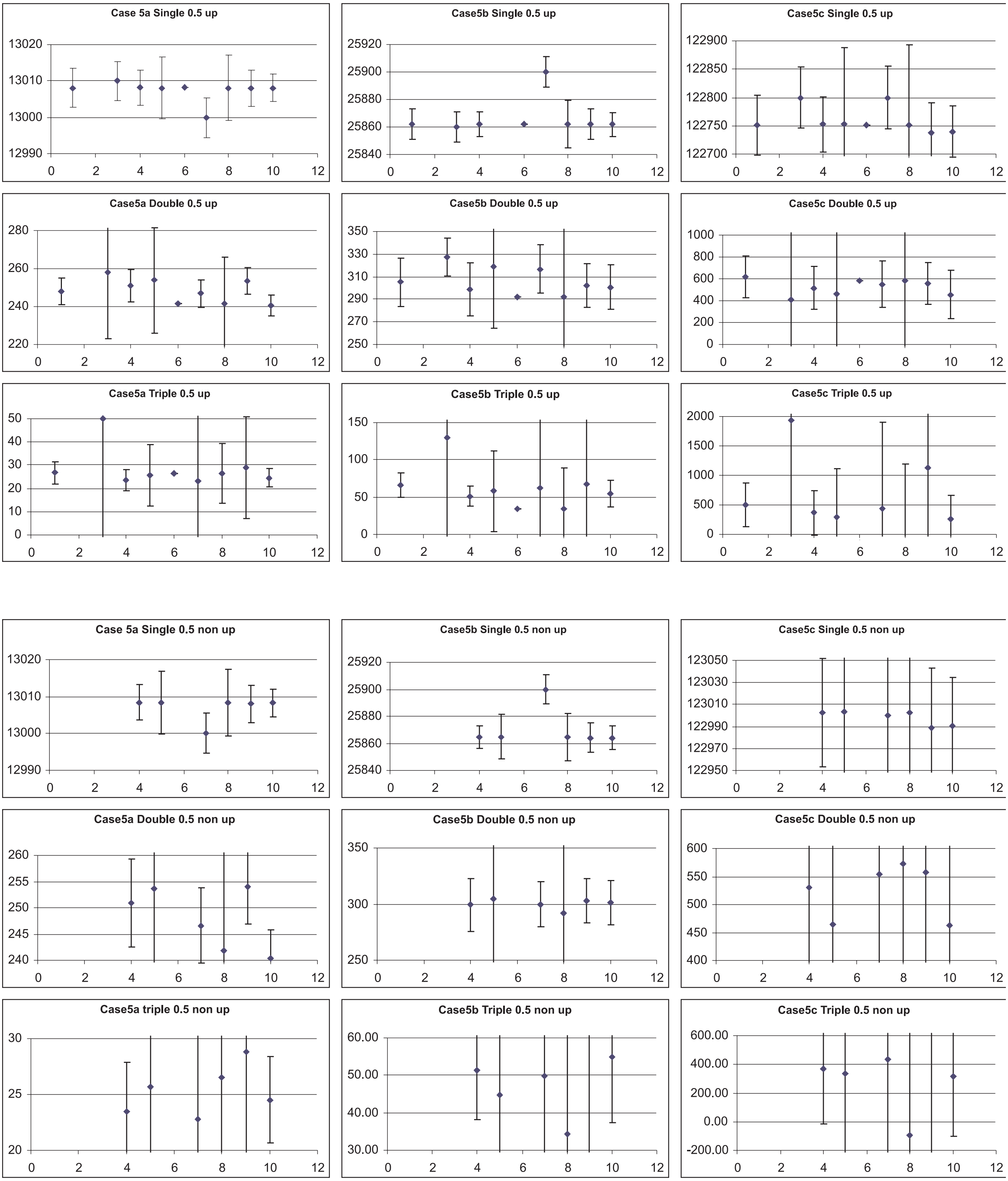

Figure B.5.2: Results for case 5 at 0.5 μs dead-time (updating and non-updating).

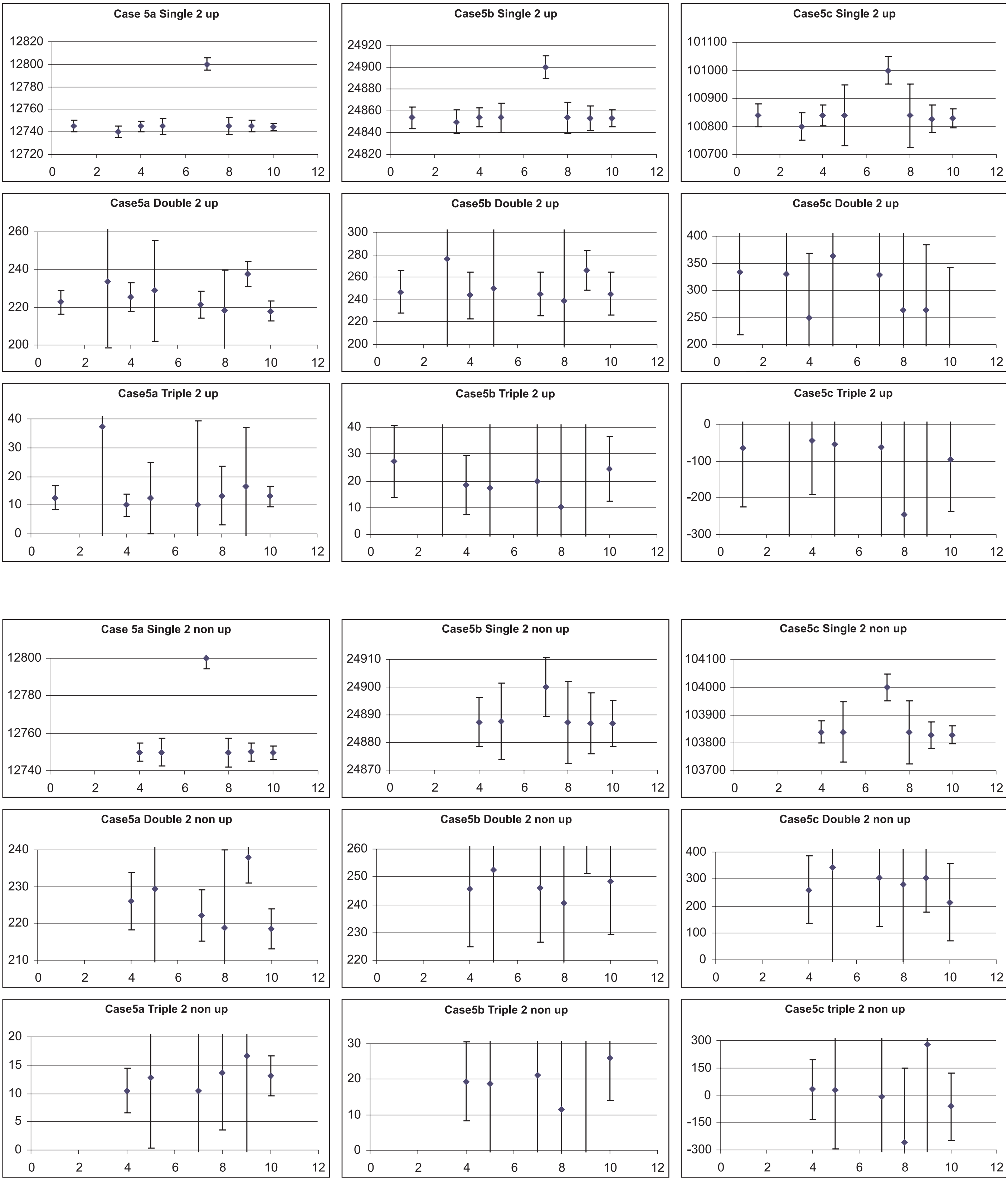

Figure B.5.3: Results for case 5 at 2 μs dead-time (updating and non-updating).

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