Non-destructive Assay in the Role of Consistency Checking

Abstract:
This paper shows quantitatively in a generic calibration context that the “forward model only approach,” i.e. comparing predicted to measured observables, has larger detection probability (DP) than the “invert to infer mass approach,” because it avoids noise amplification when inverting from NDA counts to mass.
Topics

• Inspector Verification Measurements

• Forward modelling only \( Y = \beta_0 + \beta_1 x_T + R_Y \)

• Calibration: \( Y = \beta_0 + \beta_1 x_T + R_Y \) but infer \( x \) using either \( \hat{x} = \frac{(Y_{test} - \hat{\beta}_0)}{\beta_1} \) or \( X = \alpha_0 + \alpha_1 Y_T + R_X \)

• Example: Cherenkov digital viewing device

• Conclusion
When O has declared mass and I has measured mass:

\[ O_{ij} = T_{ij} + T_{ij} B_O + T_{ij} S_{Oi} + T_{ij} R_{Oij} \]

Usually: the groups are inspection periods

\[ \text{Prob(} \text{discover 0 defects in sample of size } n) = \sum_{i=\max(0, n+r-N)}^{\min(n, r)} A_i \times B_i \]

Selection probability

\[ A_i = \frac{\binom{r}{i} \binom{N-r}{n-i}}{\binom{N}{n}} \]

Non-identification probability

\[ B_i = P(d_1 \leq 3\delta_T, d_2 \leq 3\delta_T, \ldots, d_i \leq 3\delta_T) \]

\[ \delta_T = \sqrt{\delta_R^2 + \delta_S^2} = \sqrt{\{\delta_{RO}^2 + \delta_{SI}^2\} + \{\delta_{SO}^2 + \delta_{SI}^2\}} \]

This slide assume O has declared mass and I has measured mass.
What if do not infer mass, but use operator declarations to predict detector response?
Regression (if infer mass or not)

Illustrated using \[ Y = \beta_0 + \beta_1 x_T + R_Y \] but applies for any functional form
Identification and detection probabilities

\[ Y = \beta_0 + \beta_1 x_T + R_Y \]

\[ \delta_{R_Y} = 0.07, \delta_{R_x} = 0.001 \]

DP: for a stratum of \( N = 100 \) items and a sample of \( n = 5 \) items
Zykov 2015: verify operator declarations by predicting detector data

\[ Y_{\text{true}} = \beta_0 + \beta_1 X_{\text{true}}, \text{ Y is detector data, X is measurand.} \]

\[ Y_M = Y_{\text{true}} (1 + R_Y), X_M = X_{\text{true}} (1 + R_X) \]

\[ \delta_{RX} = 0.001, \delta_{RX} = 0.05, \beta_0 = 1, \beta_1 = 0.9, n = 5 \]

\[ \beta_1 = \beta_1 \text{ (item)} \]

Note: long-term bias in calibration.
Calibration: solve for \( X \)
Regression: use \( X \) to predict \( Y \)

Note: short-term bias in regression and calibration (2 calibrations).
Example: Digital Cherenkov Viewing Device
DCVD

\[ \hat{\delta}_T = \delta_T \]
Uranium neutron collar (UNCL)

\[ Y = \frac{kX}{a_1 - a_2 kX} , \] where \( Y \) is gms \(^{235}\)U per cm

\( a_1 \) and \( a_2 \) are calibration parameters

\( k \) is a product of correction factors that adjust neutron doubles count rate \( X \) to item-, detector-, and source-specific conditions in the calibration → “errors in predictors” and associated item-specific bias → \( kX \) is noisy predictor

Excuse to be lazy: recent simulations → “ignore” errors in predictors literature.
Meas-True data spanning multiple “groups”

\[ M_{ij} = T_{ij} + T_{ii}S_i + T_{ii}R_{ij} \]

Real (O-I)/O data for UNCL method exhibiting item-specific bias

The variance of the group means is too large to be by chance, and is larger than bottom-up UQ predicts.
UNCL: observing large RSDs

\[ Y = \frac{kX}{a_1 - a_2 kX} \]

Why the large RSDs?
Calibration issues & paper describes need for better input data for model corrections

Example from paper:
9 X values: 111.1, 132.0, 149.7, 158.8, 164.1, 173.4, 176.0, 180.8, 186.5.
Corresponding: 9 \(^{235}\text{U}\) values are: 16.20, 21.89, 27.59, 29.37, 31.15, 33.28, 34.71, 36.84, 38.98.

Apply a single noise factor and to introduce noise due to departure from calibration conditions. In each simulation, cross validation was applied, in which 6 of the 9 (X, \(^{235}\text{U}\)) pairs were randomly selected to calibrate, and the other 3 (X, \(^{235}\text{U}\)) pairs were used to test. Varying amount of random error in \(k\) was applied, ranging from 1 to 5% RSD, which represented the aggregate effect of errors in \(k\).

Case 1: a total relative RSD of 0.03 in the inputs (in training and testing), the observed RSD in predicted \(^{235}\text{U}\) was approximately 0.10, which is in approximate agreement with top-down evaluation of 4 groups of 5 paired (O,I) values.

Generally, if there are different error magnitudes in testing than in training, then bias can be introduced in the estimated \(^{235}\text{U}\) and this bias contributed to the RSD of 0.15 in case 2. The message for bottom-up UQ is: understand and quantify the error sources in testing data so that training data for calibration can be obtained that has similar error RSDs.
Summary

Larger DP for regression than calibration.

If opt for calibration, best option (among 4 options) is: be lazy, ignore errors-in-predictors literature, and fit

\[ X = \alpha_0 + \alpha_1 Y_T + R_X \]

If opt for regression/consistency checking, still need appropriate forward modelling, but avoiding “inverting” the forward model except to estimate the DP.