Optimal Sampling Plans for Inventory Verification of Spent Fuel Ponds

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Abstract:

Spent fuel unloaded from the reactor core of Light Water Reactors is usually stored in the spent fuel pond of the reactor. The IAEA and EURATOM have a number of different instruments in their verification instrument portfolios to verify spent fuel assemblies in the spent fuel pond. Depending on the situation, e.g., the type of the different fuel assemblies' strata and the accessibility for future re-verification, different requirements for the verification exist.

Once spent fuel has been loaded into dry storage casks for transport and intermediate storage, it becomes difficult-to-access. The IAEA requires that nuclear material prior to its becoming difficult-to-access must be verified using sampling plans that provide a high detection probability for a possible diversion of nuclear material from the spent fuel assemblies.

The paper discusses how to set up optimal sampling plans depending on the verification instruments, the assumed detection capabilities of these verification instruments, and the presumed diversion strategies.

Keywords: sampling plans; detection probability; inventory verification; spent fuel ponds; difficult-to-access;

1. Introduction

Spent fuel assemblies (SFAs) from Light Water Reactors (LWRs) are usually stored in the spent fuel pond of the reactor. Typically, physical inventory verification (PIV), interim inspections or inspections to verify the transfers of spent fuel to dry storage, where the spent fuel is no longer accessible for verification, are carried out. As especially the verification of spent fuel prior to transfers to dry storage require a high detection probability (DP) for possible diversions and therefore also a substantial amount of inspection effort, IAEA and EURATOM aim to optimize the effectiveness and efficiency of these verifications.

The IAEA and EURATOM have a set of instruments that can be used for the verification of spent fuel assemblies. In this paper we focus on the following three instruments: The Improved Cerenkov Viewing Device (ICVD), the Digital Cerenkov Viewing Device (DCVD), and the Passive Gamma Emission Tomography System (PGET). Each of these verification instruments has an associated detection capability and specific time to perform a measurement. Depending on different diversion assumptions and nuclear material strata, optimal sampling plans can be set up based on the different characteristics of the verification instruments.

The paper is structured as follows: Section 2 introduces the verification instruments together with their detection capabilities and the types of LWR assemblies considered in this paper. Also, the estimated net measurement time to carry out the measurements for an experienced and inexperienced inspector for any of the three instruments is given. Section 3 deals with the probabilistic aspects for finding optimal sampling plans where approaches from [1], [2] and [3] are modified and tailored to the situation discussed here. In section 4 optimal sampling plans are determined for four examples where two types of LWR assemblies and two different required detection probabilities are assumed. Section 5 deals with non-equal diversion scenarios which are usually not addressed in common safeguards literature. It is shown that the achieved DP depends on the order the sampling is performed. In section 6 the case of two classes of SFAs in one spent fuel pond is treated and issues in finding optimal sampling plans are discussed. Section 7 contains the derivations of the DP formula and section 8 points to future research activities and gives an outlook.

2. Instruments for verification of spent fuel assemblies

In this paper we assume that the inventory of the spent fuel pond does not increase, which would be, e.g., the case after the final shutdown of the reactor. In this situation we consider applying the ICVD, the DCVD or the PGET to verify the content of the spent fuel pond. This section describes briefly the function and use of the three measurement instruments.

2.1 ICVD and DCVD

Both the ICVD and DCVD are inspector tools imaging the Cerenkov light emitted from irradiated nuclear fuel assemblies in spent fuel ponds as described in [4], [5], [6].

The ICVD and DCVD systems have been approved by the IAEA for gross defect verification in order to verify the presence of spent fuel. They both observe the Cerenkov light glow from above a storage pool. They are optimized for ultraviolet radiation and are capable of operating with facility lights turned on. When aligned vertically above the tops of fuel assemblies the verification instruments can distinguish irradiated fuel items from non-fuel items.

The ICVD is used for the qualitative verification (Yes/No decision) of irradiated nuclear fuel stored under water. It allows the inspectors to conclude that the observed object is an irradiated fuel assembly. ICVD does not allow recording the measurement results for further investigation. In case of inconclusiveness of the measurement results the measurement has to be repeated on-site.

The DCVD is further approved by the IAEA to detect partial defects. The camera is connected to a computer that uses specialized software to analyse the image. To carry out the measurements of SFAs stored in a pond the DCVD has to be placed at the fuel assembly-loading machine. Based on experiences a DCVD measurement campaign of spent fuel assemblies stored underwater may take up to one week depending on the number of SFAs to be verified.

In general, ICVD and DCVD have a minor impairment on the facility operation as there is no need for fuel movement or contact of the instrument with the potentially contaminated water of the spent fuel pond.

2.2 PGET

The PGET is approved by the IAEA for verification of spent nuclear fuel assemblies stored under water; see [7]. Fission products contained in SFAs emit gamma radiation, which is detected by two collimated CdZnTe detectors. The PGET system is assembled on a rotary baseplate inside a watertight torus shaped enclosure. To carry out the verification measurements the system is positioned under water on the top of an empty rack or on a special tripod on the bottom of the pond. In a next step a SFA is moved and placed in the center of the enclosure and held stationary to perform a measurement. Thus, compared to the ICVD and DCVD additional time to place the SFA in the PGET is needed.

2.3 LWR assemblies and detection capabilities of ICVD, DCVD and PGET

This paper considers two different types of LWR spent fuel such as fuel assemblies from pressurized water reactors (PWR) and boiling water reactors (BWR); see, e.g., [8]. PWRs are operated with fuel assemblies based on a square lattice arrangement characterized by the number of rods they contain, typically, 17 × 17 in current designs with for example 250 fuel pins per assembly. BWRs also use fuel assemblies, which are designed as a square lattice, with rods geometries ranging from 6 × 6 to 10 × 10 containing for example 96 fuel pins.

Each of the three verification measurement instruments has an associated detection capability: What is the probability (the so-called identification probability $p_{instrument}$) that the instrument will identify a falsified SFA as falsified, if a certain percentage of material has been removed from the SFA (defect size)? According to [9], the identification probabilities can be assumed to be step functions taking values zero and one:

$$
\begin{aligned}
p_{ICVD} &= \begin{cases} 1 & \textit{if } 100\% \textit{ of the pins have been removed} \\ 0 & \textit{otherwise} \end{cases} \\
p_{DCVD} &= \begin{cases} 1 & \textit{if } 30\% \textit{ of the pins or more have been removed} \\ 0 & \textit{otherwise} \end{cases} \\
p_{PGET} &= \begin{cases} 1 & \textit{if } 0.38\% \textit{ of the pins or more have been removed} \\ 0 & \textit{otherwise} \end{cases}
\end{aligned}
\tag{1}
$$

Six comments: First, the identification probabilities in Eq. (1) are based on field trials on multiple fuel types for a range of burnups, cooling times, and number of SFAs measured; see [9]. Second, Eq. (1) indicates that the measurement errors are not normally distributed, as it is usually assumed; see [3]. Third, more complex identification probabilities than in Eq. (1) could be considered if needed. The approach in this paper, however, would have to be modified. Fourth, Eq. (1) quantifies for our purposes the terms gross, partial and bias defect which are usually defined only qualitatively; see 10.7 in [6]. Fifth, because most of the modern LWR assembly designs allow for the exchange of single fuel pins for defective rods, Eq. (1) covers the diversion of single pins. Sixth, a replacement of pins with dummy pins, which contain material leading to a similar signal as a spent fuel pin, is not considered in this paper.

Estimates of the net measurement time for one SFA for any of the three instruments are listed in Table 1, based on personal communication [10]. Note that the set-up time for all three instruments is equal concerning the positioning of the fuel element handling machine; in addition, PGET requires 2 more minutes for the SFA placement.

	ICVD	DCVD	PGET
experienced inspector	3 seconds	1 minute	5 minutes measurement plus 2 minutes placement procedure of the SFA into the PGET
inexperienced inspector	7 seconds	2 minute	

Table 1: Estimated net measurement times for an experience/inexperienced inspector for the three instruments.

Recent experiences show, that the DCVD net measurement time of an experienced inspector is about 2 minutes per SFA instead of 1 minute depending, e.g., on the quality of water, the pond conditions, the light-contrast of the image, and the burnup of the SFAs.

3. One class of SFAs: The detection probability

This section deals with the probabilistic aspects for finding optimal sampling plans, where approaches from [1], [2] and [3] are modified and tailored to the situation discussed here.

Let N be the number of SFAs in the spent fuel pond, L be the number of fuel pins per SFA, $\bar{x}_{Pu}$ be the average amount of plutonium (Pu) per SFA, and SQ be the significant quantity (SQ = 8 [kg] for Pu), i.e. "the approximate amount of nuclear material for which the possibility of manufacturing a nuclear explosive device cannot be excluded."; see 3.14 in [6]. We focus in this paper on the diversion of Pu, because it should be more attractive to a diverter than low enriched uranium, as low enriched uranium 1) requires additional enrichment to become weapon usable material and 2) the number of fuel pins to be removed to get a significant quantity of 75 [kg] is usually much higher compared to acquiring a significant quantity of 8 [kg] for Pu.

IAEA sampling plans are usually based on the equal diversion hypothesis (see [3]) which means that each falsified item is falsified by the same amount of nuclear material. In the context discussed here, this hypothesis means that the diverter removes r_{pin} pins from a certain number of SFAs (items). For example: 10 pins are removed from 20 SFAs, but not: 4 pins are removed from 21 SFAs and 30 pins from 10 SFAs (see section 5). The number r_{pin} ranges from the smallest possible number of removed pins r_{min} (defined below) up to the maximum number of removed pins L.

How many SFAs have to be falsified if r_{pin} pins are removed from each of them? Because the diverter wants to acquire one significant quantity (1 SQ), r_{pin} and the respective number of falsified SFAs $r_{SFA} = r_{SFA}\left(r_{pin}\right)$ have to fulfil the inequality

$$\frac{\bar{x}_{Pu}}{L} \times r_{pin} \times r_{SFA}\left(r_{pin}\right) \geq SQ,$$

which yields, assuming that the diverter will not falsify more SFAs than necessary,

$$r_{SFA}\left(r_{pin}\right) = \left\lceil \frac{SQ \times L}{\bar{x}_{Pu}} \frac{1}{r_{pin}} \right\rceil, \qquad (2)$$

where the ceiling $\lceil a \rceil$ of a real number a is the smallest integer not less than a. Because the number of falsified SFAs must be smaller or equal than the total number of SFAs of the spent fuel pond, i.e. $r_{SFA}\left(r_{pin}\right) \leq N$ for all admissible r_{pin}, the minimum number of removed pins r_{min} is, using Eq. (2), the smallest integer fulfilling $\left\lceil SQ \times L / \bar{x}_{Pu} / r_{min} \right\rceil \leq N$, i.e.

$$r_{min} = \left\lceil \frac{SQ \times L}{N \bar{x}_{Pu}} \right\rceil.$$

Because it is assumed that the diverter can acquire 1 SQ from the SFAs in the spent fuel pond, we must have $SQ \leq N\bar{x}_{Pu}$, which yields, using Eq. (2), $r_{SFA}(L) = \left\lceil SQ / \bar{x}_{Pu} \right\rceil \leq \lceil N \rceil = N$, i.e. the existence of r_{min} can be assured. Therefore, the set of diversion strategies is assumed to be

$$X := \{r_{min}, r_{min}+1, \ldots, L\}, \qquad (3)$$

and $r_{SFA}\left(r_{pin}\right)$ according to Eq. (2) defines for any $r_{pin} \in X$ the number of falsified SFAs. Eq. (2) is illustrated in Table 2 for L = 96, $\bar{x}_{Pu}$ = 2 [kg] and SQ = 8 [kg] yielding r_{min} = 1 if $N \geq 384$.

Table 2 illustrates two effects: First, different r_{pin} values may lead to the same number of falsified SFAs. Thus, a rational diverter removes 1, 2, …, 23, 24, 26, 28, 30, 32, 35, 39, 43, 48, 55, 64, 77 or 96 pins, or in general:

$$\tilde{X} := \left\{ \begin{matrix} r_{pin} \in X : r_{SFA}\left(r_{pin}-1\right) > r_{SFA}\left(r_{pin}\right) \\ for\ all \quad r_{pin} = r_{min}+1 \ldots, L \end{matrix} \right\}.$$

Because of the properties of the DP as a function of r_{pin} (see below), only the three values $r_{pin} \in \left\{\lceil 0.3L \rceil - 1,\ L-1, L\right\}$ need to be considered for finding an appropriate

r_{pin}	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$r_{SFA}\left(r_{pin}\right)$	384	192	128	96	77	64	55	48	43	39	35	32	30	28

15	16	17	18	19	20	21	22	23	24,25	26,27	28,29	30,31	32,33,34
26	24	23	22	21	20	19	18	17	16	15	14	13	12

35…38	39…42	43…47	48…54	55…63	64…76	77…95	96
11	10	9	8	7	6	5	4

Table 2: Pairs $\left(r_{pin}, r_{SFA}\left(r_{pin}\right)\right)$ that achieve 1 SQ = 8 [kg].

verification sampling plan; the specific nature of the set of diversion strategies does not matter and thus, we use the set X (not $\widetilde{X}$) as set of diversion strategies. Second, not any number r_{SFA} (e.g., $r_{SFA}(r_{pin}) = 25$) can be achieved under the equal diversion hypothesis and the pin removal scenario.

Let $p_{instrument}(r_{pin})$ be the instruments' identification probability in case r_{pin} pins are removed from an SFA. Then Eq. (1) yields for all $L \le 100/0.38 = 263$

$$\begin{aligned} p_{ICVD}(r_{pin}) &= \begin{cases} 1 & for \quad r_{pin} = L \\ 0 & for \quad 1 \le r_{pin} \le L-1 \end{cases} \\ p_{DCVD}(r_{pin}) &= \begin{cases} 1 & for \quad \lceil 0.3L \rceil \le r_{pin} \le L \\ 0 & for \quad 1 \le r_{pin} \le \lceil 0.3L \rceil - 1 \end{cases} \\ p_{PGET}(r_{pin}) &= 1 \quad for \quad 1 \le r_{pin} \le L. \end{aligned} \tag{4}$$

Sampling plans are usually based on the DP. In the context analysed in this paper we are interested in the detection of the diversion of 1 SQ of Pu from the spent fuel pond by performing item by item tests. This means here that the number of reported pins in an SFA is compared to the number of identified pins in that SFA. For that purpose, the inspector verifies n_1 SFAs with the ICVD, n_2 with the DCVD, and n_3 with the PGET, where per verified SFA only one measurement instrument is applied. Using the probability mass function of a hypergeometric distributed random variable, see, e.g., [1] or [11], the detection probability $DP(N,n_1,n_2,n_3,r_{pin})$ is, using Eq. (4), given by

$$DP(N,n_1,n_2,n_3,r_{pin}) = \begin{cases} 1 - \dfrac{\binom{N - r_{SFA}(r_{pin})}{n_3}}{\binom{N}{n_3}} & for \quad r_{\min} \le r_{pin} \le \lceil 0.3L \rceil - 1 \\ 1 - \dfrac{\binom{N - r_{SFA}(r_{pin})}{n_2 + n_3}}{\binom{N}{n_2 + n_3}} & for \quad \lceil 0.3L \rceil \le r_{pin} \le L-1 \\ 1 - \dfrac{\binom{N - r_{SFA}(r_{pin})}{n_1 + n_2 + n_3}}{\binom{N}{n_1 + n_2 + n_3}} & for \quad r_{pin} = L, \end{cases} \tag{5}$$

keeping in mind that $\binom{a}{b} := 0$ for $b > a$ where a and b are integers, and where $r_{SFA}(r_{pin})$ is given by Eq. (2). The derivation of Eq. (5) is done in section 7. Note that because the identification capability of the measurement instruments is modelled as a Yes/No decision, i.e. an attribute test is performed, the DP degenerates to a selection probability. Nevertheless, we still call the expression in Eq. (5) DP because this term is commonly used; see [3] for a detailed discussion. Also note, that in the three regions of r_{pin}-values in Eq. (5) only the numbers n_i are utilized for which the respective measurement instrument(s) yield(s) the identification probability of one.

We seek for a sampling plan (n_1, n_2, n_3) that achieves the required DP $1-\beta_{req}$ (set by the IAEA and/or EURATOM) independent of the actual diversion strategy, i.e.

$$DP(N,n_1,n_2,n_3,r_{pin}) \ge 1-\beta_{req} \ for\ all\ r_{pin} \in X. \tag{6}$$

A sampling plan is called optimal, if it fulfils inequality (6), and if it minimizes the number of SFAs to be verified with the most expensive/time-consuming method (i.e. n_3), then minimizes the number of SFAs to be verified with the second most expensive/time-consuming method (i.e. n_2), and finally, minimizes the number of SFAs to be verified with the least expensive/time-consuming method (i.e. n_1), i.e. the lexicographic optimization criterion is applied; see [12].

Because the DP as defined by Eq. (5) is a monotone decreasing function of r_{pin}, i.e. $DP(N,n_1, n_2,n_3,r_{pin}) \ge DP(N,n_1, n_2,n_3,r_{pin}+1)$ for r_{pin} and r_{pin} + 1 from the same region, inequality (6) has to be valid for the three values $r_{pin} \in \{\lceil 0.3L \rceil - 1, L-1, L\}$. Thus, the sampling plan (n_1, n_2, n_3) achieves the required DP $1-\beta_{req}$ if and only if

$$\max\left\{ \frac{\binom{N - r_{SFA}(\lceil 0.3L \rceil - 1)}{n_3}}{\binom{N}{n_3}}, \frac{\binom{N - r_{SFA}(L-1)}{n_2 + n_3}}{\binom{N}{n_2 + n_3}}, \frac{\binom{N - r_{SFA}(L)}{n_1 + n_2 + n_3}}{\binom{N}{n_1 + n_2 + n_3}} \right\} \le \beta_{req}. \tag{7}$$

Because for any N, any $1 \le r < N$ and any $1 \le r \le N - r$ we have, see, e.g., [1] or [3],

$$\frac{\binom{N-r}{n}}{\binom{N}{n}} = \prod_{j=0}^{r-1}\left(1 - \frac{n}{N-j}\right) \le \prod_{j=0}^{r-1}\left(1 - \frac{n}{N}\right) = \left(1 - \frac{n}{N}\right)^r,$$

the sampling plan

$$\begin{aligned} n_3 &= \left\lceil N\left(1 - \sqrt[r_{SFA}(\lceil 0.3L \rceil - 1)]{\beta_{req}}\right)\right\rceil, \quad n_2 = \left\lceil N\left(1 - \sqrt[r_{SFA}(L-1)]{\beta_{req}}\right)\right\rceil - n_3 \\ n_1 &= \left\lceil N\left(1 - \sqrt[r_{SFA}(L)]{\beta_{req}}\right)\right\rceil - (n_3 + n_2) \end{aligned} \tag{8}$$

fulfils inequality (7) where $r_{SFA}(r_{pin})$ is given by Eq. (2). Note that the sampling plan given by Eq. (8) does not need to be optimal because it may overestimate the sample size by up to 3 SFAs (this is a general result in the attribute sampling context; see [13]).

4. One class of SFAs: Examples

In this section optimal sampling plans are determined for the examples in Table 3 where we vary 1) the required DP of 0.5 and 0.9 (first column), and 2) the type of SFAs in the spent fuel pond (BWR and PWR; first row). All other entries in Table 3 are explained in the course of this section.

	BWR, $N = 2500$, $L = 96$, $\bar{x}_{Pu} = 2$	PWR, $N = 500$, $L = 250$, $\bar{x}_{Pu} = 9$
$1-\beta_{req} = 0.5$	*ICVD*: 74 *DCVD*: 203 *PGET*: 121 *experienced*: 17 hours *inexperienced*: 21 hours	*ICVD*: 0 *DCVD*: 170 *PGET*: 80 *experienced*: 12 hours *inexperienced*: 15 hours
$1-\beta_{req} = 0.9$	*ICVD*: 172 *DCVD*: 543 *PGET*: 379 *experienced*: 53 hours *inexperienced*: 62 hours	*ICDV*: 0 *DCVD*: 231 *PGET*: 219 *experienced*: 29 hours *inexperienced*: 33 hours

Table 3: Examples.

<table>
<tr><td>r_{pin}</td><td>1</td><td>2</td><td>…</td><td>28</td><td>29</td><td>30</td><td>…</td><td>95</td><td>96</td></tr>
<tr><td>$r_{SFA}(r_{pin})$</td><td>384</td><td>192</td><td>…</td><td>14</td><td>14</td><td>13</td><td>…</td><td>5</td><td>4</td></tr>
<tr><td>$p_{ICVD}(r_{pin})$</td><td colspan="4" rowspan="2">0</td><td colspan="4">0</td><td rowspan="3">1</td></tr>
<tr><td>$p_{DCVD}(r_{pin})$</td><td colspan="4" rowspan="2">1</td></tr>
<tr><td>$p_{PGET}(r_{pin})$</td><td colspan="4">1</td></tr>
</table>

Table 4: Pairs $\left(r_{pin}, r_{SFA}\left(r_{pin}\right)\right)$ that achieve $1\,SQ = 8$ [kg] and the identification probabilities as a function of the number r_{pin} of removed pins.

For a detailed discussion, we consider the BWR – 90 % example: The spent fuel pond contains $N = 2500$ BWR SFAs, each having $L = 96$ fuel pins and an average amount $\bar{x}_{Pu} = 2$ [kg] of Pu per SFA. Table 4 illustrates in the first and second row the pairs $\left(r_{pin}, r_{SFA}\left(r_{pin}\right)\right)$ that achieve $1\,SQ = 8$ [kg] of Pu, where $r_{SFA}\left(r_{pin}\right)$ is given by Eq. (2). The third to fifth row represents the instruments' identification probability according to Eq. (4).

Table 4 demonstrates that in case of a bias defect (i.e. only small numbers of removed pins) only the PGET is capable to identify a falsified SFA as falsified, whereas only in case of a gross defect (all pins are removed) all instruments lead to the identification probability one.

For illustration, consider the sampling plan $(n_1, n_2, n_3) = (10,65,25)$. Figure 1 shows the achieved DP given by Eq. (5) as a function of the number r_{pin} of removed pins. The two vertical lines divide the graph into three regions: in the left region ($1 \le r_{pin} \le 28$) only the PGET is capable to identify a falsified SFA as falsified, in the middle region ($29 \le r_{pin} \le 95$) both the PGET and the DCVD are capable to identify a falsified SFA as falsified, and only in the right region ($r_{pin} = L = 96$) all three instruments are successful in identifying a falsified SFA as falsified.

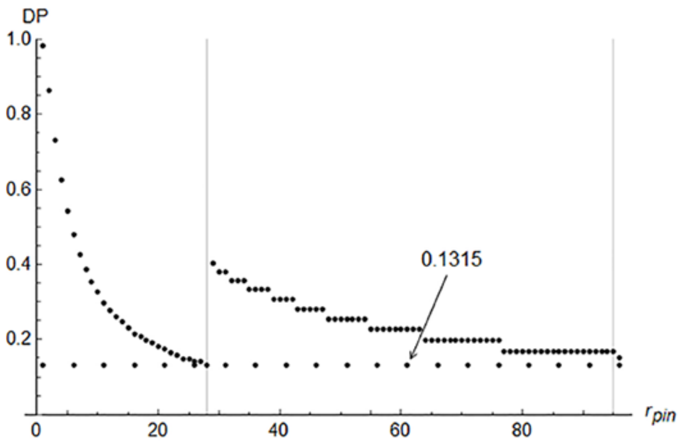

Figure 1: Achieved DP for the sampling plan $(n_1, n_2, n_3) = (10, 65, 25)$.

Three comments on Figure 1:

First, the sampling plan $(n_1, n_2, n_3) = (10,65,25)$ achieves a DP of about 0.1315 (horizontal dots) independent of the actual diversion strategy because the minimum of $DP\left(N, n_1, n_2, n_3, r_{pin}\right)$ is attained at $r_{pin} = 28$, and we obtain, using Eq. (5), that $DP\left(N, n_1, n_2, n_3, 28\right) \approx 0.1315$.

Second, the DP curve is a monotone (but not strictly) decreasing function of r_{pin} in the regions $r_{pin} = 1, \ldots, 28$ and

r_{pin} = 29, …, 95, respectively. This result is plausible, because with an increasing number of removed pins the number of falsified SFAs in the pond decreases (i.e. $r_{SFA}(r_{pin}) \geq r_{SFA}(r_{pin}+1)$), and thus, the probability to select at least one falsified SFA when $r_{SFA}(r_{pin})$ are in the pond is greater or equal than the probability to select at least one falsified SFA when $r_{SFA}(r_{pin}+1)$ are in the pond.

Third, the value of the DP is constant, e.g., for all numbers r_{pin} = 48, …,54, because they lead to the same number of falsified SFAs ($r_{SFA}(48) = \ldots = r_{SFA}(54) = 8$, see Table 2) and because in that region both the PGET and the DCVD are capable to identify a falsified SFA as falsified. In contrast, although $r_{SFA}(28) = r_{SFA}(29) = 14$ we see that $DP(N,n_1,n_2,n_3,28) < DP(N,n_1,n_2,n_3,29)$, because in case r_{pin} = 29 both the PGET and the DCVD are being used and $n_2 > 0$.

Suppose the inspector wants to achieve a required DP of (say) 0.9. How many SFAs does he need to verify? Eq. (8) implies that the sampling plan (n_1, n_2, n_3) = (172,543,380) achieves a DP of 0.9, i.e. at least 380 SFAs must be verified by the PGET. This sampling plan, however, is not optimal in the sense defined after inequality (6), because the number of PGET measurements can be reduced to 379 and the sampling plan (172,543,379) still achieves a DP of 0.9. This illustrates that the sampling plan given by Eq. (8) does not have to be optimal (see also the comment at the end of section 3). Using the net measurement times from Table 1, we see that an experienced inspector needs about 53 hours net measurement time, while an inexperienced one needs about 62 hours (times are rounded down to full hours). Both measurement times are unrealistic and thus, the optimal sampling plan (172,543,379) may be infeasible in practice. The results of the BWR-90% example are summarized in the respective field in Table 3.

The optimal sampling plans for the remaining examples in Table 3 can be obtained in the same way. Therefore, we just make two comments: First, the identification probabilities for the PWR examples are, using Eq. (4),

$$p_{ICVD}(r_{pin}) = \begin{cases} 1 & \text{for} \quad r_{pin} = 250 \\ 0 & \text{for} \quad 1 \leq r_{pin} \leq 249 \end{cases}$$

$$p_{DCVD}(r_{pin}) = \begin{cases} 1 & \text{for} \quad 75 \leq r_{pin} \leq 250 \\ 0 & \text{for} \quad 1 \leq r_{pin} \leq 74 \end{cases}$$

$$p_{PGET}(r_{pin}) = 1 \quad \text{for} \quad 1 \leq r_{pin} \leq 250.$$

Second, because Eq. (2) implies $r_{SFA}(249) = r_{SFA}(250) = 1$, the requirement $DP(N,n_1,n_2,n_3,95) \geq 1-\beta_{req}$ implies $DP(N,n_1,n_2,n_3,96) \geq 1-\beta_{req}$ for any $n_1 \geq 0$, and thus, there is no need to perform any ICVD measurement.

It has to be emphasized that the optimal sampling plans in Table 3 highly depend on the definition of the identification probabilities given by Eqs. (1) and (4).

To further illustrate the approach of section 3, we discuss two topics. First, we determine the optimal sampling plan if either "PGET and ICVD" (case 1) or "PGET and DCVD" (case 2) can be applied. Consider a spent fuel pond with N = 2000 BWR SFAs (with L = 96 pins each) and an average amount $\bar{x}_{Pu} = 0.5\,SQ = 4$ [kg] of Pu per SFA. If the required DP is 0.9, then the optimal sampling plans that achieve the DP of 0.9 are given by

$$case1: \begin{matrix} ICVD: & 296 \\ PGET: & 1072 \end{matrix} \quad , \quad case2: \begin{matrix} DCVD: & 808 \\ PGET: & 560 \end{matrix}$$

The number of PGET measurements in case 1 is considerably higher compared to case 2. This is due to the fact that in case 1 the PGET has to cover all pin removals r_{pin} with $1 \leq r_{pin} \leq 95$, where in case 2 only the region $1 \leq r_{pin} \leq 28$ has solely covered by the PGET.

Second, suppose only 20 PGET measurements can be performed, but there is no restriction on the number of ICVD measurements, i.e. the entire spent fuel pond can be verified by the ICVD. Thus, we consider the sampling plan (n_1, n_2, n_3) = (1980,0,20) that yields, using Eq. (5), an achieved DP of about 0.028, which is usually far too low in this context.

In this section only one class of SFAs has been considered which is a valid assumption if the variation (variance) of the amount of Pu amongst the SFAs in the spent fuel pond is not "too big". An example with two classes of SFAs is presented in section 6.

5. One class of SFAs: Non-equal diversion scenarios

In this section non-equal diversion scenarios, which are usually not addressed in common safeguards literature, are considered.

The sampling plans in sections 3 and 4 are based on the equal diversion hypothesis which leads to the relation between r_{pin} and $r_{SAF}(r_{pin})$ in Eq. (2). The diverter, however, does not need to falsify SFAs according to this hypothesis. For illustration, let N = 2000, L = 96 and $\bar{x}_{Pu} = 2$ [kg] and let the diverter removes 4 pins from 21 SFAs and 30 pins from 10 SFAs. Then he acquires exactly

$$\frac{2\ [kg]}{96} \times (4 \times 21 + 30 \times 10) = 8\ [kg]\ ,$$

of Pu. Using Table 4, we see that the 21 SFAs falsified by 4 pins each can only be successfully identified as falsified by the PGET, while the 10 SFAs falsified by 30 pins each can be identified as falsified by both the PGET and the DCVD.

Let us assume that the required DP is 0.5. Following the same procedure as described in section 4 for BWR-90 % example, we find the sampling plan (n_1, n_2, n_3) =

(59,162,97) to be optimal if the equal diversion hypothesis is true. What is the achieved DP if the diverter removes the pins as indicated above and if the inspector sticks to the optimal sampling plan (59,162,97)?

We first determine the achieved DP in case the sampling is performed in the order $n_3 \to n_2 \to n_1$, which means that we first sample the SFAs verified by the PGET, then we sample (from the remaining $N - n_3$ SFAs) the SFAs verified by the DCVD, and finally we sample (from the remaining $N - n_3 - n_2$ SFAs) the SFAs verified by the ICVD. The event of non-detection occurs if for the PGET measurements none of the 21 + 10 = 31 SFAs are in the sample, and for the subsequent DCVD measurements none of the 10 SFAs with 30 missing pins are in the sample. Thus, the achieved DP is

$$1 - \frac{\binom{31}{0}\binom{N-31}{n_3}}{\binom{N}{n_3}} \frac{\binom{10}{0}\binom{N-n_3-10}{n_2}}{\binom{N-n_3}{n_2}} \approx 0.91.$$

In case the sampling is done in the order $n_3 \to n_2 \to n_1$, the event of non-detection occurs if for the DCVD measurements none of the 10 SFAs with 30 missing pins but i SFAs out of the 21 SFAs with 4 missing pins (and which can only be identified with PGET) are sampled, and for the subsequent PGET measurements none of 21 – i remaining SFAs with 4 missing pins, are sampled. The achieved DP is, using the multivariate hypergeometric distribution (see [11]), given by

$$1 - \sum_{i=0}^{21} \frac{\binom{21}{i}\binom{10}{0}\binom{N-31}{n_2-i}}{\binom{N}{n_2}} \frac{\binom{21-i}{0}\binom{N-n_2-(21-i)}{n_3}}{\binom{N-n_2}{n_3}} \approx 0.85.$$

Two comments on this example: First, the achieved DP depends on the order the sampling is performed: The order $n_3 \to n_2 \to n_1$ leads – at least in this example – to a higher DP than the order $n_2 \to n_3 \to n_1$. Whether this is true in general has to be investigated. In section 7 it is shown that under the equal diversion hypothesis the order of sampling does not have any influence on the DP.

Second, the achieved DP is – for both orders considered above and using the optimal sampling plan (59,162,97) – higher than 0.5 which was used for finding (59,162,97) under the equal diversion hypothesis. Therefore, one can ask whether this hypothesis is a worst case in the sense that the sampling plan found under the equal diversion hypothesis also achieves the required DP for all non-equal diversion scenarios, i.e. in case the diverter does not remove pins according to the equal diversion hypothesis.

6. Two classes of SFAs: Example

We now consider two classes of BWR (with L = 96 pins each) SFAs with N_1 = 217 and N_2 = 297 SFAs per class in the same spent fuel pond, e.g., one class of SFAs with high burn-up and one class of SFAs with very low burn-up. The average amounts of Pu are assumed to be $\bar{x}_1 = 1.5$ [kg] and $\bar{x}_2 = 3$ [kg] of Pu per SFA in class i = 1,2. Again, the diverter wants to remove pins from SFAs to acquire 8 [kg] of Pu. In contrast to sections 3 and 4 he can now get the material from both classes: If m_i, i = 1,2, denotes the nuclear material mass diverted from class i, then we have $m_1 + m_2 \geq 8$. To keep the example manageable, we only take special values of (m_1, m_2) into account:

$$M := \{(8,0),(7,1),(6,2),(5,3),(4,4),(3,5),(2,6),(1,7),(0,8)\}. \quad (9)$$

Because $8 \leq \bar{x}_i N_i$ for i = 1,2, we have $(8,0),(0,8) \in M$. Within each class the diverter is – as before – assumed to remove $r_{pin,i}$ from $r_{SFA}(m_i, r_{pin,i})$ SFAs of class i, i = 1,2, where, using Eq. (2),

$$r_{SFA,i}(m_i, r_{pin,i}) = \left\lceil \frac{m_i \times 96}{\bar{x}_i} \frac{1}{r_{pin,i}} \right\rceil. \quad (10)$$

Let $n_{1,j}$ resp. $n_{2,j}$, j = 1,2,3, denote the sample size in class 1 and 2, where $n_{1,1}$ resp. $n_{2,1}$ SFAs are verified by the ICVD, $n_{1,2}$ resp. $n_{2,2}$ by the DCVD, and $n_{1,3}$ resp. $n_{2,3}$ by the PGET, i.e. the first index indicates the class and the second one the measurement method. The identification probabilities are given in Table 4.

Using Eq. (3), the sets $X_1(m_1)$ and $X_2(m_2)$ of diversion strategies in the first resp. second class is for i = 1,2 given by

$$X_i(m_i) = \left\{ \left\lceil \frac{m_i \times L}{\bar{x}_i N_i} \right\rceil, \ldots, L \right\}.$$

The overall (over both classes together) DP is, for any $(m_1, m_2) \in M$ and for any $(r_{pin,1}, r_{pin,2}) \in X_1(m_1) \times X_2(m_2)$, given by

$$\begin{aligned} & DP(N_1, N_2, n_{1,1}, n_{1,2}, n_{1,3}, n_{2,1}, n_{2,2}, n_{2,3}, m_1, r_{pin,1}, m_2, r_{pin,2}) \quad (11) \\ = \; & 1 - (1 - DP(N_1, n_{1,1}, n_{1,2}, n_{1,3}, m_1, r_{pin,1}))(1 - DP(N_2, n_{2,1}, n_{2,2}, \\ & n_{2,3}, m_2, r_{pin,2})), \end{aligned}$$

where $DP(N_1, n_{1,1}, n_{1,2}, n_{1,3}, m_1, r_{pin,1})$ and $DP(N_2, n_{2,1}, n_{2,2}, n_{2,3}, m_2, r_{pin,2})$ are defined by Eq. (5) in which $r_{SFA,i}(m_i, r_{pin,i})$ of Eq. (10) is used.

As before we are interested in sampling plans $(n_{1,1}, n_{1,2}, n_{1,3})$ and $(n_{2,1}, n_{2,2}, n_{2,3})$ that achieve the required DP $1-\beta_{req}$ independent of the actual diversion strategy. Because the diverter may acquire 1 SQ from one of the classes only (see the left and right element in the set M in Eq. (9)), the approach is section 3 implies that $(n_{1,1}, n_{1,2}, n_{1,3})$ and $(n_{2,1}, n_{2,2}, n_{2,3})$ have to fulfil inequality (7) for each class separately. This gives for $\beta_{req} = 0.1$ the optimal sampling plans

$$(n_{1,1}, n_{1,2}, n_{1,3}) = (0,45,24) \text{ and } (n_{2,1}, n_{2,2}, n_{2,3}) = (0,98,61) \quad (12)$$

for each class individually. Does the sampling plan given by Eq. (12) achieves the overall DP of 0.9 for all diversion strategies $(m_1,m_2)\in M$ or only for $(m_1,m_2)=(0,8)$ and $(m_1,m_2)=(8,0)$? I.e. we need to check whether the achieved overall DP is at least 0.9 for all pairs $(m_1,m_2)\in M$, i.e. whether

$$\min_{(m_1,m_2)\in M}\ \min_{(r_{pin,1},r_{pin,2})\in X_1(m_1)\times X_2(m_2)} DP\left(N_1,N_2,n_{1,1},n_{1,2},n_{1,3},n_{2,1},n_{2,2},n_{2,3},m_1,r_{pin,1},m_2,r_{pin,2}\right) \quad (13)$$

is at least 0.9. According the multiplicative structure of the overall DP in Eq. (11), the minimization of the overall DP over the set $(r_{pin,1},r_{pin,2})\in X_1(m_1)\times X_2(m_2)$ is equivalent to minimizing the individual DPs $DP(N_1, n_{1,1}, n_{1,2}, n_{1,3}, m_1, r_{pin,1})$ and $DP(N_2, n_{2,1}, n_{2,2}, n_{2,3}, m_2, r_{pin,2})$ over $X_1(m_1)$ and $X_2(m_2)$, respectively.

Consider the pair $(m_1,m_2)=(2,6)$. Then Eqs. (5) and (12) imply (rounded to the fourth digit after the dot)

$$DP\left(N_1,n_{1,1},n_{1,2},n_{1,3},2,r_{pin,1}\right)=\begin{cases}0.4467 & r_{pin,1}=28\\ 0.5358 \quad for & r_{pin,1}=95\\ 0.5358 & r_{pin,1}=96\end{cases}$$

and

$$DP\left(N_2,n_{2,1},n_{2,2},n_{2,3},6,r_{pin,2}\right)=\begin{cases}0.8037 & r_{pin,2}=28\\ 0.9009 \quad for & r_{pin,2}=95\\ 0.7849 & r_{pin,2}=96,\end{cases}$$

and thus, we get for the achieved overall DP by Eq. (11) and the comment after Eq. (13)

$$1-(1-0.4467)\times(1-0.7849)=0.881<0.9,$$

i.e. the sampling plan given by Eq. (12) does not achieve the required overall DP of 0.9. If we increase in Eq. (12) the ICVD measurements by 1 verification in each class, and the DCVD and PGET measurements by 3 verifications in each class, i.e. if we consider the sampling plan

$$\left(n_{1,1},n_{1,2},n_{1,3}\right)=\left(1,48,27\right)\ and\ \left(n_{2,1},n_{2,2},n_{2,3}\right)=\left(1,101,64\right), \quad (14)$$

then the overall DP of 0.9 is achieved for all diversion strategies $(m_1,m_2)\in M$.

Two remarks: First, the sampling plan given by Eq. (14) is most likely not optimal, i.e. a sampling plan with shorter net measurement times might be found using more sophisticated methods other than just shifting sample sizes by a constant. Second, the sampling plan given by Eq. (14) achieves the overall DP of 0.9 for all diversion strategies $(m_1,m_2)\in M$ but not necessarily for all diversion strategies of a more refined set which includes M as a subset, such as

$$\{(8,0),(7.5,0.5),(7,1),\ldots,(1,7),(0.5,7.5),(0,8)\}$$

in which, in contrast to Eq. (9), an incremental step of 0.5 [kg] is used. Indeed, the sampling plan given by Eq. (14) yields an achieved overall DP of about 0.898 for $(m_1,m_2)=(4.5,3.5)$.

7. Derivations

To derive Eq. (5), we need to model the way the random sampling is performed: Quite generally, the following orders are possible: $n_3\to n_2\to n_1$, $n_3\to n_1\to n_2$, $n_2\to n_1\to n_3$, $n_2\to n_3\to n_1$, $n_1\to n_2\to n_3$ and $n_1\to n_3\to n_2$ ($n_3\to n_2\to n_1$ means: first sample the SFAs that are verified by the PGET, then sample the SFAs verified by the DCVD, and finally sample the SFAs verified by the ICVD) and the possibility to choose $n_1+n_2+n_3$ SFAs from the population of SFAs and then distributed them to the verification methods i, $i=1,2,3$. We claim that Eq. (5) is true independent of the chosen order, and prove this statement for the cases: 1) $n_3\to n_2\to n_1$ and 2) $n_2\to n_3\to n_1$. Although the proof in case 1) is rather obvious, we present it here in order to show the differences between cases 1) and 2).

Ad 1): If $1\le r_{pin}\le\lceil 0.3L\rceil-1$, then non-detection occurs if and only if none of the $r_{SFA}(r_{pin})$ falsified SFAs are sampled for PGET verifications. The DCVD and ICVD measurements do not influence the DP in this case. Thus, we get with $r_{SFA}:=r_{SFA}(r_{pin})$

$$1-\frac{\binom{r_{SFA}}{0}\binom{N-r_{SFA}}{n_3}}{\binom{N}{n_3}}=1-\frac{\binom{N-r_{SFA}}{n_3}}{\binom{N}{n_3}} \quad (15)$$

which is the first equation in Eq. (5). If $\lceil 0.3L\rceil\le r_{pin}\le L-1$, then non-detection occurs if and only if none of the $r_{SFA}(r_{pin})$ falsified SFAs are sampled for PGET or DCVD verifications. The ICVD measurements do not influence the DP in this case. Thus, we have

$$1-\frac{\binom{r_{SFA}}{0}\binom{N-r_{SFA}}{n_3}}{\binom{N}{n_3}}\frac{\binom{r_{SFA}}{0}\binom{N-n_3-r_{SFA}}{n_2}}{\binom{N-n_3}{n_2}}=1-\frac{\binom{N-r_{SFA}}{n_2+n_3}}{\binom{N}{n_2+n_3}} \quad (16)$$

where the equal sign can be shown by expanding the binomial coefficients. If $r_{pin}=L$, then we get in analogy to Eq. (16)

$$1-\frac{\binom{N-r_{SFA}}{n_3}}{\binom{N}{n_3}}\frac{\binom{N-n_3-r_{SFA}}{n_2}}{\binom{N-n_3}{n_2}}\frac{\binom{N-(n_3+n_2)-r_{SFA}}{n_1}}{\binom{N-(n_3+n_2)}{n_1}}=1-\frac{\binom{N-r_{SFA}}{n_1+n_2+n_3}}{\binom{N}{n_1+n_2+n_3}} \quad (17)$$

i.e. the third equation in Eq. (5).

Ad 2): If $1\le r_{pin}\le\lceil 0.3L\rceil-1$, then non-detection occurs 1) if any SFA is sampled for DVCD verifications (i.e. even falsified SFAs can be sampled, because they can only be identified as falsified by the PGET), and 2) if none of the remaining falsified SFAs are sampled for PGET verifications. Again, the ICVD measurements do not have an impact on the DP in this case. Thus, we obtain for the DP with $r_{SFA} := r_{SFA}(r_{pin})$

$$1-\sum_{i=Max(0,n_2+n_3-(N-r_{SFA}))}^{Min(r_{SFA},n_2)} \frac{\binom{r_{SFA}}{i}\binom{N-r_{SFA}}{n_2-i}\binom{r_{SFA}-i}{0}\binom{N-n_2-(r_{SFA}-i)}{n_3}}{\binom{N}{n_2}\binom{N-n_2}{n_3}}. \quad (18)$$

Note that the lower bound of the sum in Eq. (18) is due to the fact that $N-r_{SFA} \geq n_2 - i$ and $N-n_2-(r_{SFA}-i) \geq n_3$, which is equivalent to $i \geq n_2-(N-r_{SFA})$ and $i \geq n_2+n_3-(N-r_{SFA})$.

Eqs. (15) and (18) demonstrate the influence of the order the sampling is done on the DP formula. As announced, however, we show that both DP formulae are equivalent.

Expanding the binomial coefficients, the sum expression in Eq. (18) simplifies to

$$\binom{N}{r_{SFA}}^{-1} \sum_{i=Max(0,n_2+n_3-(N-r_{SFA}))}^{Min(r_{SFA},n_2)} \binom{n_2}{i}\binom{N-(n_2+n_3)}{r_{SFA}-i}. \quad (19)$$

Using the identity

$$\sum_{k=Max(0,m-b)}^{Min(m,a)} \binom{a}{k}\binom{b}{m-k} = \binom{a+b}{m}$$

for any positive integers *a*, *b* and *m*, see [11], we finally get from Eq. (19), using the substitutions $a \to n_2$, $b \to N-(n_2+n_3)$ and $m \to r_{SFA}$,

$$\binom{N}{r_{SFA}}^{-1} \sum_{i=Max(0,n_2+n_3-(N-r_{SFA}))}^{Min(r_{SFA},n_2)} \binom{n_2}{i}\binom{N-(n_2+n_3)}{r_{SFA}-i} = \frac{\binom{N-n_3}{r_{SFA}}}{\binom{N}{r_{SFA}}} = \frac{\binom{N-r_{SFA}}{n_3}}{\binom{N}{n_3}},$$

i.e. the first line of Eq. (5) and Eq. (15). If $\lceil 0.3L \rceil \leq r_{pin} \leq L-1$, then the non-detection event occurs if and only if none of the $r_{SFA}(r_{pin})$ falsified SFAs are sampled for the PGET and DCVD verifications. Thus, its probability is given by Eq. (16) changing $n_3 \to n_2$ and $n_2 \to n_3$. The case $r_{pin} = L$ is given by Eq. (17) again with the replacements $n_3 \to n_2$ and $n_2 \to n_3$.

8. Future work and outlook

The following four topics could be examined in future research activities: First, a sensitivity analysis could be carried out that studies the consequences of the instruments identification thresholds on optimal sampling plans. For example: The ICVD identification threshold could be lower down from 100% to 50%, i.e. the identification probability is one if and only if "50% of the pins or more have been removed"; see Eq. (1). For the DCVD, a reduction of the 30% threshold down to 10% could be considered.

Second, as expanded at the end of section 5, it should be investigated whether optimal sampling plans obtained under the equal diversion hypothesis assure the required DP for all non-equal diversion scenarios. Also the influence of the order the sampling is performed on the DP needs to be investigated.

Third, in case of a non-homogeneous population of SFAs in a spent fuel pond the question of the appropriate number of classes of SFAs, which are characterized by different average amounts of Pu, has to be addressed. This number will depend on the SFAs masses and their variation (variance). A related topic has already been investigated in [14].

Fourth, the example in section 6 calls for an efficient algorithm for the determination of optimal sampling plans in case more than one class of SFAs are stored in the spent fuel pond.

In a future scenario the use of robots could support the inspectorates to confirm the presence of spent fuel in a pond. In this light a nuclear focused robotics challenge co-hosted by the IAEA took place in November 2017 in Australia. The aim of this challenge was to attach an ICVD inside a small robotized floating platform, which autonomously carries out the verification measurements by moving across the surface of the pond; see [15]. The consequences of using robots on the sample sizes can be studied as soon as it becomes clear which inspection activities can be carried through or supported by robots and what the robots' identification capabilities are.

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10. Disclaimer

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